
Duality, Non-Geometry and its Phenomenology

Rui Sun



München 2016

Duality, Non-Geometry and its Phenomenology

Rui Sun

Dissertation
an der Fakultät für Physik
der Ludwig-Maximilians-Universität
München

vorgelegt von
Rui Sun
aus Heilongjiang, China

München, September 6, 2016

Erstgutachter: PD. Dr. Ralph Blumenhangen

Zweitgutachter: Prof. Dr. Dieter Lüst

Tag der mündlichen Prüfung: November 7, 2016

路漫漫其修遠兮，
吾將上下而求索。

Zusammenfassung

Die vorliegende Doktorarbeit beschäftigt sich mit verschiedenen Aspekten von nicht-geometrischen Hintergründen, wie z.B. der Herleitung von “Double Field Theory (DFT)”, Anwendungen bezüglich geometrischer Deformationen sowie Stringphänomenologie. Grundlegende Konzepte der Stringtheorie, T-Dualität, DFT und Supergravitation werden ebenso erläutert.

Der erste Teil dieser Arbeit beginnt mit einer Einführung in generalisierte Geometrie als wichtigstes mathematisches Konzept. Außerdem wird die Notation von Lie-Algebroiden ausführlich wiederholt, die durch Anwendung auf geometrische Stringhintergründe zu nicht-geometrischen Hintergründen führen. Nicht-geometrische Hintergrundkonfigurationen, d.h. Metrik und Bi-Vektor Feld, werden durch Kotangentialbündel beschrieben, wohingegen Tangentialbündel geometrische Hintergründe charakterisieren. Wir analysieren generalisierte Konfigurationen in heterotischer DFT und definieren Eichvektoren in nicht-geometrischen Hintergründen als T-duale von Eichfeldern auf geometrischen Hintergründen. Für jede Feldredefinition, gegeben durch eine $O(D, D+n)$ Transformation, ist die Struktur der erhaltenen Wirkung der heterotischen Supergravitation bestimmt durch die Differentialgeometrie eines zugehörigen Lie-Algebroids.

Der zweite Teil dieser Arbeit ist der Anwendung von nicht-geometrischen Hintergründen auf nicht-assoziative Raumzeitdeformationen gewidmet. Dabei wird die Struktur möglicher mathematischer Deformationen detailliert in der Fluss-Formulierung von DFT analysiert und es wird gezeigt, dass in der effektiven DFT-Wirkung Assoziativität “on-shell” nicht verletzt ist. Solche nicht-assoziativen Deformationen, die aber “on-shell” Assoziativität erhalten, werden sowohl für den “strong-” als auch den schwächeren “closure-constraint” diskutiert.

Der dritte Teil dieser Doktorarbeit behandelt Anwendungen von nicht-geometrischen Hintergründen auf Stringphänomenologie. Insbesondere stellen nicht-geometrische “flux-scaling” Vakua vielversprechende Ausgangspunkte dar, um Axion-Monodromie-Inflation durch F-Term Skalarpotentiale zu realisieren. Es wird gezeigt, dass diese Vakua vom Typ Minkowski oder de Sitter sein können falls man $\overline{D3}$ -Branen oder D-Terme hinzufügt, die geometrische und nicht-geometrische Flüsse enthalten. Zudem wird untersucht, inwiefern diese “uplifted” nicht-supersymmetrischen Modelle auf konsistente Art und Weise Axion-Monodromie-Inflation ermöglichen. Unter Hinzunahme von rationalen Flüssen, konstruieren wir explizite Beispiele mit korrekter Massen-Hierarchie.

Abschließend werden alle Projekte zu nicht-geometrischen Hintergründen nochmal zusammengefasst und ein Ausblick auf zukünftige Forschung gegeben.

Abstract

In this thesis, we present some aspects of non-geometric backgrounds, e.g. how non-geometric backgrounds are derived in Double Field Theory (DFT), as well as their applications to deformations of geometry and string phenomenology. Basics of string theory, T-duality, DFT and supergravity concepts are reviewed.

The first part of the thesis begins by introducing generalized geometry as the main mathematical framework. In addition, the notion of Lie-algebroid is reviewed in detail and the non-geometric string backgrounds is approached via Lie-algebroid by acting on the dual geometric string backgrounds. The non-geometric background configuration, i.e. a metric and a bivector field, is described by the cotangent bundle, while the geometric background is described by the tangent bundle. We study the generalized configuration in heterotic DFT framework and introduce gauge vectors on non-geometric backgrounds as the T-duals of gauge fields on the geometric backgrounds. Q - and R -fluxes on the non-geometric backgrounds are redefined in terms of metric, bivector and gauge vector fields. For every field redefinition specified by an $O(D, D+n)$ transformation, the structure of the resulting heterotic supergravity action is governed by the differential geometry of a corresponding Lie algebroid.

The second part of the thesis is devoted to the application of non-geometric backgrounds to the non-associative deformation of space-time geometry. In the flux formulation of DFT, the structure of possible mathematical non-associative deformations is analyzed in detail. We show that on-shell there is no violation of associativity in the effective DFT action. For imposing either the strong or the (weaker-)closure constraint, we discuss two possible non-associative deformations of DFT which feature two different ways of how on-shell associativity can still be kept.

In the third part of the thesis, we give examples of non-geometric backgrounds application to string phenomenology. The non-geometric flux-scaling vacua provide promising starting points to realize axion monodromy inflation via F-term scalar potential. We show that these vacua can be uplifted to Minkowski and de Sitter ones by adding an $\overline{D3}$ -brane or a D-term containing geometric and non-geometric fluxes. These uplifted non-supersymmetric models are analyzed with respect to realize axion monodromy inflation consistently. Admitting rational values of the fluxes, we construct examples with the required hierarchy of mass scales.

In the last part of the thesis, the projects related to non-geometric backgrounds are summarized and an outlook is given.

Acknowledgments

First of all, I would like to express my special gratitude to my supervisor Priv.-Doz. Ralph Blumenhagen for the chance to work with him, expert advices and encouragement all through my PhD. I benefited a lot from each discussion we had for the projects and out of projects. He not only shared his striking broad knowledge without any reservation but also his ideas, intuitions and way of research. He has made an enormous effort to equip me with the knowledge and skills for doing research. He also encouraged me constantly to attend scientific conferences/discussions and present our work. I am also very grateful to our director Prof. Dieter Lüst for his continuous support, encouragement and providing a most pleasant and productive work environment in our string theory group. I also learned a lot from his questions and discussions raised up in group seminars and journal clubs. It helped me very much to broaden my knowledge on various aspects of physics and encouraged me to explore in research. Furthermore, I am in great debt to Prof. Klaus Altmann for his interests in our research and support.

I thank Henk Bart, Federico Bonetti, Andreas Deser, Johanna Erdmenger, Anamaria Font, Michael Fuchs, Xin Gao, Prieslei Goulart, Sebastian Greiner, Thomas Grimm, Daniel Junghans, Andreas Kapfer, Severin Lüst, Emanuel Malek, Kilian Mayer, Nina Miekley, Alexander Millar, Pramod Shukla, Tom Pugh, Erik Plauschinn, Christian Schmid, Piotr Witkowski and Yigit Yargic for many valuable discussions and creating a friendly atmosphere. I would like to thank Taoli Cheng, Mario Flory, Daniela Herschmann, Jan Keitel, Wai Yeung Lam, Max-Niklas Newrzella, Jana Pietsch and Charlotte Sleight for accompanying me all through my PhD both academically and privately. I especially thank Andre Betz, Yanyan Bu, Cesar Damian, Inaki Garcia-Etxebarria, Falk Haßler, Diego Regalado, Felix Rennecke and Irene Valenzuela for patiently helping me with all my questions without any reservation. I thank Yuta Sekiguchi for collaborating with his most effort. Although we haven't finished our project yet, I truly hope we will finish it soon in the near future. I am particularly grateful to my Bavarian group brother and officemate Florian Wolf for raising up many interesting discussions, for his support, being wise and friend. I thank Pierre Corvilain for creating a relaxing office atmosphere, teaching us F-theory and bringing many spontaneous lectures to our office, in particular from Diego and Irene. Furthermore, I thank all from Max-Planck-Institute for Physics and Ludwig Maximilian University of Munich for being around and encouraging.

I also want to thank the organizers of the conferences I have been to. Thank you all for the opportunities to attend the conferences and give talks. In addition, I thank Federico Carta, Aitor Landete, Liam McAllister, Miguel Montero, Ander Retolaza, Wieland Staessens, Fengjun Xu, Gianluca Zoccarato for sharing their knowledge generously during the conferences and helping me to prepare talks. It is very nice to know you all.

Last but not least, I would like to thank my parents, sister and friends for their unconditional love, continuous support and encouragement.

Contents

Zusammenfassung	III
Abstract	IV
Acknowledgments	VII
I Introduction	1
1 Introduction	3
2 String Theory	11
2.1 Basics of string theory	11
2.2 Duality	14
2.2.1 T-duality of closed bosonic strings	14
2.2.2 T-duality of superstrings	15
2.2.3 S-duality	17
2.3 Double field theory	19
2.3.1 Review of double field theory	19
2.3.2 Heterotic double field theory	23
2.4 Exceptional field theory	27
2.4.1 $SL(5)$ exceptional field theory	28
3 String Compactification	31
3.1 Type IIB string compactification	31
3.1.1 Type IIB effective action	31
3.1.2 String compactifications	32
3.2 Calabi-Yau orientifold compactification	33
3.2.1 Calabi-Yau manifolds	33
3.2.2 Orientifold projections	34
3.2.3 Orientifold compactification with fluxes	37

II	String Applications	43
4	Non-geometry in Heterotic DFT and EFT	45
4.1	Non-geometric backgrounds of heterotic DFT	46
4.1.1	T-duality of a constant gauge flux background	46
4.1.2	The fluxes of heterotic DFT	50
4.1.3	Comment on S-duality	52
4.2	A Lie algebroid for heterotic field redefinitions	52
4.2.1	Lie algebroids	53
4.2.2	$O(D, D + n)$ -induced field redefinition	54
4.2.3	The redefined heterotic action	57
4.2.4	The non-geometric frame	58
4.3	Lie algebroid and $SL(5)$ transformation	60
4.4	Summary and discussion	61
5	Non-associative Deformations of Geometry	63
5.1	Non-associativity in physics	66
5.1.1	Non-associativity for magnetic monopoles	67
5.1.2	Open string with non-associative star product	68
5.2	Flux formulation of DFT	70
5.3	Non-associative deformations of DFT	72
5.3.1	A tri-product for $\check{\mathcal{F}}^{ABC}$	73
5.3.2	A tri-product for \mathcal{F}_{ABC}	76
5.3.3	Non-associativity in heterotic DFT	80
5.4	Summary and discussion	80
6	String Phenomenology with Non-Geometries	83
6.1	Uplifting to de Sitter	85
6.1.1	Uplift via $\overline{D3}$ -brane	85
6.1.2	D-term uplift	90
6.2	Axion monodromy inflation	93
6.2.1	Effective field theory approach	93
6.2.2	Numerical analysis of inflation	95

CONTENTS

6.3	Summary and discussion	100
III	Conclusions and Outlook	101
7	Conclusions and Outlook	103
7.1	Summary of research projects	103
7.2	Outlook	105
A	K tri-product	107
B	The heterotic Buscher rules	109
C	Non-holonomic fluxes for heterotic DFT	111
	Bibliography	113

CONTENTS

Part I

Introduction

1

Introduction

Physics has been extraordinarily developed from its origin to the current era. It provides a powerful framework to describe the phenomena from subatomic distances to the size of the observable universe. On the particle physics side, the Standard Model successfully describes the known elementary particles and their interactions. There are two groups of distinguished particles, fermions which constitute the matter content of our universe and bosons which mediate interactions between fermions. Three kinds of weak bosons W^+, W^-, Z mediate the weak interaction. Eight massless gluons act as the gauge bosons for the strong force between quarks, analogous to the exchange of photons in the electromagnetic force between two charged particles. The electromagnetic and weak interactions can be merged into one single electroweak interaction and described by the gauge group $SU(2) \times U(1)_Y$ which is spontaneously broken by the Higgs mechanism to $U(1)_{em}$, the gauge group of Quantum electrodynamics. During the electroweak symmetry breaking, the bosons interact with the Higgs field and obtain mass. The phrase “Higgs mechanism” refers specifically to the generation of masses for the weak gauge bosons W^+, W^- and Z . In 2012, the Large Hadron Collider at CERN announced results consistent with the Higgs particle, confirming a Higgs-like particle exists [1, 2], which is further elucidated in 2013 [3, 4] regards to how the Higgs mechanism takes place in nature. The vacuum expectation value of the Higgs field fixes the electroweak scale at around 246 GeV where electromagnetic and weak interactions are unified. At higher energy scale, a Grand Unified Theory is a model in particle physics, in which the three gauge interactions of the Standard Model (the electromagnetic, weak, and strong interactions) are merged into one single force. This Grand unified interaction is characterized by one larger gauge symmetry, several force carriers, but one unified coupling constant. If Grand Unification is realized in nature, there is the possibility of a grand unification epoch in the early universe in which the above three fundamental forces are not yet distinct. This is predicted to be at energy scale of 10^{16} GeV. At even higher energy scale, namely near Planck scale 1.22×10^{19} GeV, the four kinds of fundamental interactions, including gravitational force, are assumed to be unified and build a Theory of Everything.

In recent years, a great deal of achievements from experimental cosmology provided us with new insights into the history of the universe. The observation of Cosmic Microwave Background (CMB) radiation shows that the average mean temperature is the same in all

directions with an almost scale invariant spectrum of tiny perturbations. This gave strong hints for an inflationary epoch in the early universe, during which the universe experienced a fast accelerated expansion to guarantee the homogeneous and isotropic property of CMB. In the current era, CMB shows that the universe is still under accelerated expansion which predicts that an unusual dark energy exists. The standard model of cosmology shows that the total mass-energy of the universe contains 4.9 % ordinary matter, 26.8 % dark matter and 68.3 % dark energy. The most accepted hypothesis on the form for dark matter is that it is composed by Weakly Interacting Massive Particles (WIMPs) that interact only through gravity and weak interaction. The positive expansion rate of the universe confirms a positive cosmological constant, and thus shows our universe is a de Sitter vacuum solution. In 2014, BICEP2 published a large tensor-to-scale ratio $r \approx 0.2$ which motivated the study of large field inflation [5]. Due to the lyth bound,

$$\frac{\Delta\phi}{M_{\text{Pl}}} = \mathcal{O}(1) \sqrt{\frac{r}{0.01}}, \quad (1.0.1)$$

a large tensor-to-scalar value r implies a field range $\Delta\phi > M_{\text{Pl}}$ which corresponds to large field inflation. In 2015, the PLANCK collaboration announced a preciser tensor-to-scalar ratio $r < 0.113$ which gives more restriction to large field inflation models [6]. Weak Gravity Conjecture (WGC) suggested to rule out the theory of extranatural inflation [7] from the beginning. Recently, there has been renewed interest regards to how the WGC constrains large field inflation [8–17]. Therein, inflation models of axion with shift symmetries are also constrained by WGC. However, axion monodromy models, with the symmetry broken in a controlled way by inducing monodromy, allow sub-planckian field range and can still play as good inflation models.

A most fundamental theory, like Theory of Everything, which can describe both microscopic and large scale is in a great demand. In the past few decades a great deal of research effort has been devoted to possible embeddings of the Standard Model and cosmology. String theory is raised up as a good candidate of Theory of Everything. It describes particles with a one-dimensional object, a string. Strings can be open and closed. Different observed particles arise from excitation modes of these strings. The first excitation mode of a closed string leads to a particle of spin 2 which plays the role of graviton. The energy scale in which the extended dimensions of the string live, is known as string scale. It is typically close to the Planck scale¹. In addition, string theory has demonstrated to accommodate non-abelian gauge symmetries and chiral fermions, allowing for the possible embedding of the Standard Model.

The string theory spacetime dimension is fixed by the internal self-consistency of the theory. In the case of supersymmetric string theories, accounting for the presence of fermions, the spacetime dimension has to be ten. This implies that there are six extra spatial dimensions in addition to the four dimensional spacetime in which we live. The extra dimensions must be compact and sufficiently small to be undetectable at the scales currently

¹The quantum effects of gravity become important when it is closer to Planck scale.

accessible by experiments. To connect string theory with the four dimensional physical world, one needs to compactify the theory from ten to four dimensions. The compactified dimensions are called internal dimensions, while the remaining dimensions are external dimensions. Depending on the manifolds structure of the internal dimensions, there are plenty of ways for compactifications that lead to different vacuum solutions on string landscapes. Each vacuum corresponds to a possible universe described by string theory. These vacuum solutions are parameterized by so-called moduli, which highly depend on the size and shape of the internal dimensions.

String phenomenology is developed to find the compactifications corresponding to our universe. On the particle physics side, many properties of the standard model depend purely on the local aspects of the compactification, such as the mass of particles and the gauge interactions. An extension of string theory, F-theory, also played an very important role in this exploration. In the last decades, there have been big developments in the possible embeddings of the standard model in string theory and F-theory using local models. On the other hand, in the large scale direction, many properties of our universe depend on global aspects, such as the cosmological constant responsible for the acceleration of the universe. Great achievements have been made in the exploration of string phenomenology on different aspects. Much work remains to be done for a possible complete compactification which works both microscopically and in large scale. For example, fine tuning models for positive cosmological constant (i.e. de Sitter universe) and inflation models are still under investigation. Many related fields, e.g. Moduli Stabilization [18], Calabi-Yau compactifications [19], KKLt scenario [20] and nilpotent fields [21,22] have been intensively studied along this direction.

The stringy realization of axion monodromy inflation has become an active area of research after the inception in [23–26], (see e.g. [27, 28] for reviews). In [29], the axion responsible for inflation was identified with a deformation modulus of a D7-brane, whereas in [30, 31] the axion was related to the B -field from the NS-NS sector integrated over a non-contractible internal two cycle. In [32] non-geometric fluxes were included in the effective theory identifying the Kähler modulus with the inflaton. Other scenarios realize axion inflation in warped resolved conifolds [33], which suffers from a too small string scale for a large axion decay constant [34]. The case of chaotic inflation with axionic-like fields considering the backreaction of the heaviest moduli has been worked out in [35]. Another attempt to embed chaotic inflation is [36] where the axion was identified with either a Wilson line or the position modulus of a D-brane containing the MSSM. In the framework of F-theory [37], an axion-like field serves as inflaton for natural inflation. Special points in the moduli space for which the complex structure moduli can drive axion monodromy inflation were investigated in [38].

In the past years, potential realizations of dS vacua in string theory have been intensively studied from different perspectives [20, 39–48]. Both analytical and numerical approaches have been followed to construct metastable dS vacua. Moreover, no-go theorems have been derived in the context of type II [49–57] and heterotic [58–60] superstring

theories. One of the loopholes of these no-go theorems is the restriction of the fluxes to those visible in supergravity. However, by arguments based on T-duality [61,62] and the developments in generalized geometry and Double Field Theory (DFT) [63–67] it has become clear that there might also exist so-called non-geometric fluxes. For instance, the *STU*-models [68–72] were analyzed in much detail for realizations of dS vacua by introducing T- and S-dual non-geometric fluxes.

On the formal aspects of string theory, the on-shell solutions of the two-dimensional field theory on the world-volume of the probe string are provided by Conformal Field Theories (CFTs) with critical central charge. String theory models incorporate the particles and fundamental forces of nature in one unified theory, while itself is constituted of several versions of string theory, open and closed bosonic string (26-dimensional), type I superstring, type IIA/IIB superstring and heterotic $SO(32)/E_8 \times E_8$ superstring. The five types of superstring theories can be considered as different limit of a more fundamental theory and be unified to a so-called 11-dimensional M-theory. With S-duality manifested in the version of $SL(2, \mathbb{Z})$, type IIB superstring theory can be generalized to 12-dimensional F-theory. The above string theories are linked by T- and S-dualities as it is shown in Figure 1.1. Supergravity is considered as the low energy limit of superstring theory.

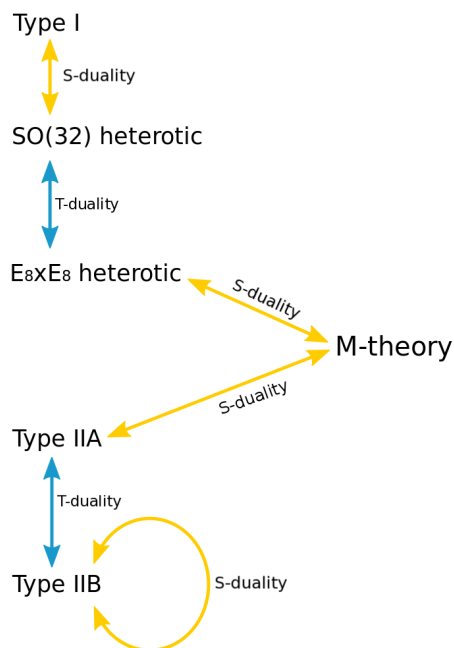


Figure 1.1: Dualities in String Theory

With T-duality manifested into a global $O(D, D)$ symmetry, supergravity can be generalized to DFT. Besides the standard geometric framework, DFT also incorporates non-geometric backgrounds as the T-dual of geometric ones. The understanding and description of non-geometric string backgrounds have been under investigation in the recent

years [61, 73–75]. The classic example is to perform successive T-dualities via the Buscher rules [76, 77] applied to a flat closed string background with constant H -flux [61]. This led to a flux background chain

$$H_{abc} \rightarrow F^a{}_{bc} \rightarrow Q_c{}^{ab} \rightarrow R^{abc} \quad (1.0.2)$$

where the Q -flux background is globally non-geometric and the R -flux background is even locally non-geometric. It was shown that these non-geometric backgrounds are expressed in a so-called non-geometric frame with $(\tilde{g}_{ij}, \beta^{ij})$ instead of (g_{ij}, B_{ij}) in the standard geometric frame. The new metric and the bi-vector are related to the geometric frame via a field redefinition.

In order to properly describe such backgrounds, one needs to go beyond the usual effective supergravity description of string theory. As we mentioned, one approach is to develop a theory which is manifestly invariant under T-duality, namely DFT, where the two frames and even the coordinates themselves are extended to a doubled space by introducing winding coordinates for non-geometric frame. A frame-like formulation was worked out in [78, 79] and further developed in [66, 80]. Later, using string field theory, an equivalent generalized metric formulation was found in [81–83]. Furthermore, DFT not only features a global $O(D, D)$ symmetry but also enhanced the local symmetries according to the winding dependence. For recent reviews of DFT we refer to [63, 64, 84]. Another alternative approach to study the non-geometric frame is generalized geometry [85–88], where one extends the tangent bundle of a manifold such that diffeomorphisms and B -field gauge transformations can be described in a single geometric framework with only standard coordinates x^i . Furthermore, the metric, Kalb-Ramond B -field and the bi-vector β -field are unified in a generalized metric on the generalized bundle $TM \oplus T^*M^2$. Based on the understanding of non-geometric backgrounds, in [65, 89, 90] the authors derived the supergravity action in the non-geometric frame, which was named as β -supergravity. In [91, 92] a general structure of an $O(D, D)$ induced field redefinitions was clarified in the framework of generalized geometry. This connected the fields in the non-geometric frame with the ones in the geometric frame. They showed that for each such field redefinition, one can associate a corresponding Lie algebroid so that the redefined supergravity action is governed by the differential geometry of the Lie algebroid.

A natural generalization of bosonic DFT is heterotic DFT [78, 79, 93, 94], where the latter also includes the gauge fields present in the heterotic superstring theory and extends the T-duality transformation accordingly. In generalized geometry the heterotic string was also discussed in [95–99]. For abelian gauge fields this generalization is formally straightforward extending the global symmetry group from $O(D, D)$ to $O(D, D + n)$. For every gauge field A^α a new coordinate y^α is introduced, thus extending also the generalized metric so that it includes the gauge fields. The main relations of DFT remain unchanged so that the action still has the same form as for bosonic DFT. The abelian heterotic DFT can be

²Standard metric and B -field are parametrized in the tangent bundle T , while dual metric and β -field belong to the cotangent bundle T^* .

gauged which also allows the description of non-abelian gauge groups [94, 100]. However, in this process the global symmetry group breaks to $O(D, D)$. It was observed in [101, 102] that, in contrast to bosonic DFT, the action of T-duality gives the Buscher rules including α' corrections. In the same work, a suggestion has been made how heterotic DFT can be further generalized to also accommodate the leading order gravitational α' corrections, including e.g. the well known Chern-Simons terms involving the spin-connection. There has been quite some interest on how to incorporate such α' corrections in the framework of generalized geometry [103, 104] and DFT [101, 102, 105–107]. In [102], we also derived the α' correction in heterotic DFT framework, and discussed according to the heterotic Buscher rules.

The link between string theory and non-commutative geometry has drawn growing interest since Seiberg and Witten shed light on it [108]. For example, the effective theory for an open string moving on a D-brane becomes a non-commutative gauge theory, if a constant flux is switched on. For open strings, it has been shown that in the background of a non-constant two-form the coordinates are non-commutative and non-associative [109–111]. In closed string theory, one is necessarily also dealing with gravity and the possible target space deformations. This turned out to be deformations by a tri-product structure which can be considered as non-associative deformations of geometry [112, 113].

In Chapter 3 of the thesis, we analyze how the structure derived from a Conformal Field Theory (CFT) perspective carried over to the recently discussed (non-)geometric framework of DFT, where the geometric fluxes and the non-geometric fluxes are well-defined and unified into a doubled flux F^{ABC} . We compute the generic tri-products while the generic functions being scalars. We show that up to leading order the tri-products of modified fluxes give boundary terms when the DFT equations of motion are satisfied. However, the flux F^{ABC} is not exactly what we have in CFT, as the non-geometric R-flux is an anti-symmetric 3-vector. We show that the non-associativity is annihilated when the strong constraint is applied.

In Chapter 4, we work on getting a better understanding of the heterotic generalization of DFT. We find non-geometric backgrounds as the T-dual of constant gauge flux backgrounds, analogous to the study for the Kalb–Ramond field. In addition, we study the T-duality mapping in terms of the differential geometry of a corresponding Lie algebroid in generalized geometry framework, and how the gauge field takes part in it. We show that the resulting field redefinitions from Lie algebroid anchor mappings are consistent with those from heterotic Buscher transformations. In particular, the α' corrections are naturally incorporated within the gauge field terms. With the understanding we gain from heterotic DFT, we find that the constant non-geometric gauge J -flux background of the $E_8 \times E_8$ heterotic string can be considered as the S-dual of a type I' background with a $D8$ -brane intersecting the $O8$ -plane at an angle. Moreover, we show that the T-dual of heterotic supergravity action (the one corresponding to a non-geometric frame) can be derived with the $O(D, D + n)$ induced Lie algebroid anchor. We expect that the whole action including the fermionic terms is governed by the objects in the differential geometry

of the Lie algebroid. This includes e.g. the kinetic terms for the gravitinos and gluinos, that involve a spin-connection. Furthermore, the gravitational Chern-Simons terms follow similar rules as the gauge field terms.

In Chapter 5 of the thesis, we construct de Sitter vacua from flux compactifications and then study the large field inflation model therein. We implement the common mechanism to uplift AdS vacua to de Sitter vacua and preserve stability by introducing an $\overline{D3}$ -brane at a warped throat as in the KKLT scenario. In general, for type IIB superstring with orientifold compactifications on Calabi-Yau three-folds, with non-vanishing fluxes turned on, one gets flux induced superpotentials. Thus, the closed string moduli, namely the axio-dilation as well as the complex structure and Kähler moduli, will be stabilized by fluxes. We study type IIB orientifold compactifications with geometric and non-geometric fluxes turned on. We construct a sequence of AdS vacua via moduli stabilization from the reduced F-term scalar potentials. By implementing an $\overline{D3}$ -brane in a warped throat as in the KKLT scenario, we analyze the extra positive contributions to the scalar potential. We find tachyon-free non-supersymmetric Minkowski and de Sitter vacua. An analytical method to uplift Minkowski to de Sitter vacua by perturbing around the original vacua was constructed. As the second uplift approach, setting $h_+^{2,1} > 0$, we include the abelian gauge fields coming from the dimensional reduction of the R-R four-form on an orientifold even three-cycle of the Calabi-Yau manifold [114]. This setting introduces new contributions to the scalar potential from D-term, we show that it admits tachyon-free Minkowski/de Sitter vacua. By introducing an extra P -flux term (which is considered to be the S-dual of Q -flux) we obtain de Sitter vacuum with good inflation candidates. This procedure provides a Flux-Scaling Scenario. On the axion inflation aspects, we derive the axion potential from F-term scalar potential. Based on the de Sitter vacua we found, where the lightest state lies in axion moduli, we obtain axion inflation models with mass hierarchy fully satisfied.

This thesis is mainly based on the following publications:

- [1] T-duality, Non-geometry and Lie Algebroids in Heterotic Double Field Theory,
R. Blumenhagen and R. Sun, JHEP 1502 (2015) 097, arXiv:1411.3167 [hep-th].
- [2] Non-associative Deformations of Geometry in Double Field Theory,
R. Blumenhagen, M. Fuchs, F. Haßler, D. Lüst, and R. Sun, JHEP 1404 (2014) 141, arXiv:1411.3167 [hep-th].
- [3] The Flux-Scaling Scenario: De Sitter Uplift and Axion Inflation,
R. Blumenhagen, C. Damian, A. Font, D. Herschmann, R. Sun, Fortsch.Phys. 64 (2016) no.6-7, 536-550, arXiv:1510.01522 [hep-th].

2

String Theory

This chapter is devoted to a brief introduction of some fundamental aspects of string theory. String theory is raised up with the calling of “Theory of Everything” providing a unification of all known interactions and gravity. Here we give a brief introduction to the building blocks of string theory and the aspects most relevant for our following discussions. For a complete overview and introduction we refer to the textbooks such as [115–121].

2.1 Basics of string theory

Bosonic string theory

We begin with the action of bosonic string theory. Consider a two-dimensional worldsheet parametrized by coordinates $\sigma^\alpha = (\sigma^1, \sigma^2)$ where σ^1 denotes the space coordinate and σ^2 denotes the time coordinate. The metric on the worldsheet is denoted as $h_{\alpha\beta}$. We introduce bosonic fields $X^\mu(\sigma^1, \sigma^2)$ on the worldsheet with $\mu = 0, \dots, d-1$. These fields are mapped from the worldsheet to a d -dimensional target space whose metric is $g_{\mu\nu}$. X^μ indicates the position of a string in the target space, as illustrated in Figure 2.1. Choosing $g_{\mu\nu}$ to be the flat Minkowski metric $\eta_{\mu\nu}$, the strings are described by Polyakov action,

$$\mathcal{S} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-\det h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}, \quad (2.1.1)$$

where α' is related to the string length as $l_s = 2\pi\sqrt{\alpha'}$, and the tension of the string is denoted as $T = (2\pi\alpha')^{-1}$. The field X^μ is considered as operator from the quantization point of view. The corresponding Fourier modes are interpreted as vibration modes of the string and indicated the quantum numbers of the d -dimensional Poincaré group. Thus different excitation states of strings can be interpreted as different particles in the target space. In the spectrum of closed strings, one always finds a massless spin 2 particle which can be identified as the graviton. In this sense, string theory, especially closed string theory includes gravity. Imposing Weyl transformation on the Polyakov action, a local rescaling

is performed to the metric tensor,

$$h_{\alpha\beta}(\sigma) \rightarrow e^{-2\omega(\sigma)} h_{\alpha\beta}(\sigma) \quad (2.1.2)$$

and produces another metric in the same conformal class. By requiring the theory to be conformally invariant and anomalous free, one constraints the dimension of the target spacetime for bosonic strings to be 26. However, bosonic string theory contains a tachyon in the spectrum and thus the ground state is unstable. In addition, the bosonic string does not contain spinor-like objects of the 26-dimensional Lorentz group. This implies the absence of fermionic particles.

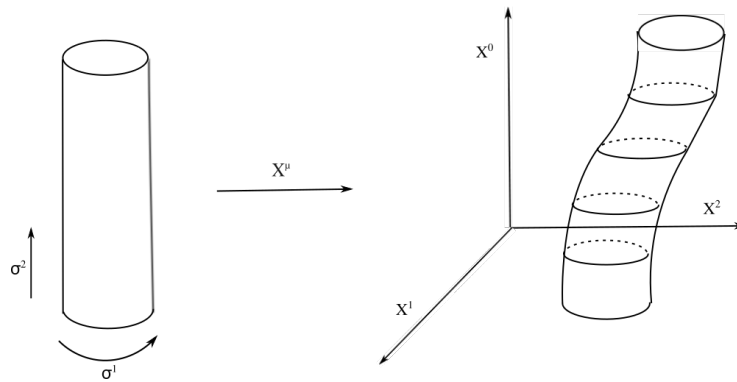


Figure 2.1: Illustration of a two-dimensional worldsheet mapped to a d -dimensional target space.

Superstring theory

Superstring theory is a supersymmetric extension of the bosonic string including fermionic degrees of freedom on the worldsheet. The Weyl anomaly cancellation fixes the spacetime dimension to be 10 for superstring theory. However, there is not only one superstring theory but several different constructions are possible. These string theories are directly or indirectly linked to each other via dualities. Well below the string scale $M_s = (\alpha')^{-\frac{1}{2}}$, these superstring theories are described by supergravity theories. Although the supergravity theory only captures the massless excitations of the string, one can study the low energy limit of string theory via supergravity. To sum up, there are five types of superstring theories.

Type I Superstring is a theory of open and closed unoriented superstrings in ten dimensions. Its low energy effective description is an $N = 1$ Super Yang-Mills theory with gauge group $SO(32)$ coupled to type I supergravity.

Type IIA and **Type IIB Superstring** are theories of closed oriented superstrings in ten dimensions. Their low energy effective descriptions are type IIA and type IIB supergravity respectively.

Heterotic Superstring is a combination of the bosonic string in the left-moving and the superstring in the right-moving sector of a closed string. Space-time is ten-dimensional and the allowed gauge groups are $SO(32)$ and $E8 \times E8$. The low energy theories are $N = 1$ Super Yang-Mills theories coupled to type I heterotic supergravity.

Apart from the standard constructions of 10-dimensional superstring theories, there is 11-dimensional **M-theory** as T-duality manifested unification of the above five types of superstring theories. Furthermore, with S-duality manifested type IIB superstring theory can be generalized to 12-dimensional **F-theory**. Regards to T- and S-dualities, we will give detailed introduction in the later part of this section.

D-brane

In string theory, apart from closed and open strings, there is also another fundamental object called D-brane. D-branes are a class of extended objects upon which open strings end with Dirichlet boundary conditions. D-branes are typically classified by their spatial dimension p , so that a D-brane can be denoted as Dp -brane. A D0-brane is a single point, a D1-brane is a line, a D2-brane is a plane, and a $D(d-1)$ -brane fills the highest-dimensional space considered in string theory. The endpoints of the open string can be confined to one single D-brane or two separated D-branes (the open string stretched between them) as it is shown in Figure 2.2.

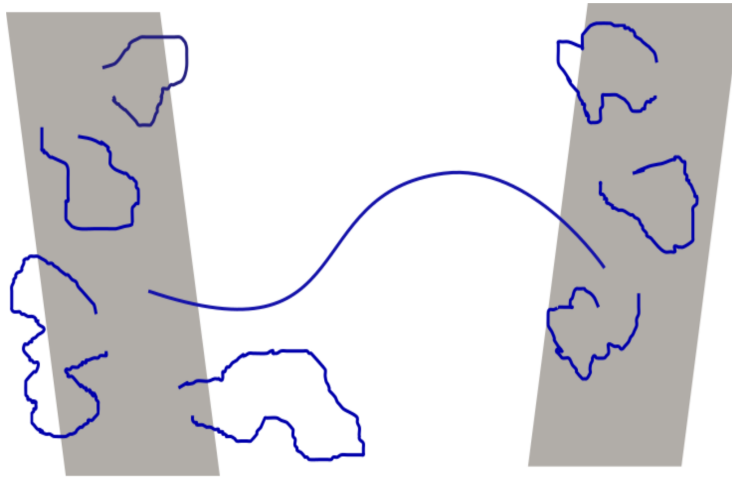


Figure 2.2: Illustration of open strings are confined to D-branes.

For the open strings with both endpoints on the same brane, the dynamics of the massless degrees of freedom is described by a pure Super Yang-Mills Theory living on the world-volume of the D-brane. For a stack of D-branes, the corresponding gauge group is $U(N)$ if there are N D-branes. Consequently, gauge theories have a geometric origin in string theory. As open strings stretch between different stacks of D-branes, if the open

strings are localized at the intersections locus of intersecting branes, it is possible to obtain chiral matter. More concretely, for two intersecting stacks of N_1 and N_2 D-branes, chiral matter transforms in bi-fundamental representations of the gauge group $U(N_1) \times U(N_2)$ which is counted by topological invariant.

In general, D-branes are allowed to intersect multiple times. Each topologically invariant intersection will give rise to one copy of chiral matter transforming in bi-fundamental representations. It provides gauge groups, chiral matter and family replication. Furthermore, the open strings can join to form closed strings, so a theory of open strings naturally comes up with a closed string sector interpreting graviton. Therefore, in string theory, gauge theories and gravity are closely related.

2.2 Duality

With the fundamental objects of string theory introduced in the last section, now we introduce the dualities which link different types of string theories. One of the most important perturbative dualities in string theory is T-duality. We start with the discussion of T-duality in bosonic string theory. Under T-duality transformations, closed bosonic strings transform into closed strings of the same type in the T-dual geometry. The situation for open strings is different because there are two types of boundary conditions that can be imposed at the ends of open strings, namely Neumann type and Dirichlet type boundary conditions. The Dirichlet boundary conditions appear in the equivalent T-dual reformulation of Neumann type ones.

2.2.1 T-duality of closed bosonic strings

In order to introduce T-duality, we start with the simplest example, namely the bosonic string with one of the 25 spatial directions compactified on a circle of radius R . We take periodic boundary condition for the compactified 25th direction. The coordinate $x^{25}(\sigma, \tau)$, $0 \leq \sigma \leq 2\pi$, maps the closed string onto the spatial circle $0 \leq x^{25} \leq 2\pi R$. Thus the closed string is modified to

$$x^{25}(\sigma + 2\pi, \tau) = x^{25}(\sigma, \tau) + 2\pi RL, \quad L \in \mathbb{Z}, \quad (2.2.1)$$

where L is the winding number. It indicates the number of times the string winds around the circle while its sign encodes the direction. The term $2\pi RL$ gives rise to strings which are closed on the circle S^1 . After quantization this leads to new states called winding states.

In more detail, the mode expansion for $x^{25}(\sigma, \tau)$ respect to (2.2.1) reads

$$x^{25}(\sigma, \tau) = x^{25} + \alpha' p^{25} \tau + LR\sigma + osc. \quad (2.2.2)$$

x^{25} and p^{25} obey the usual commutation relation $[x^{25}, p^{25}] = i$. The momentum p^{25} generates translations of x^{25} . Single valuedness of the wave function $e^{ip^{25}x^{25}}$ restricts the allowed internal momenta to discrete values $p^{25} = \frac{M}{R}$, $M \in \mathbb{Z}$. The quantized momentum states are called Kaluza-Klein modes. We split x^{25} into left and right movers

$$\begin{aligned} x_R^{25}(\tau - \sigma) &= \frac{1}{2}(x^{25} - c) + \frac{\alpha'}{2}\left(\frac{M}{R} - \frac{LR}{\alpha'}\right)(\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha_n^{25} e^{-in(\tau - \sigma)}, \\ x_L^{25}(\tau + \sigma) &= \frac{1}{2}(x^{25} + c) + \frac{\alpha'}{2}\left(\frac{M}{R} + \frac{LR}{\alpha'}\right)(\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \bar{\alpha}_n^{25} e^{-in(\tau + \sigma)}. \end{aligned} \quad (2.2.3)$$

The mass operator receives contributions from winding states

$$\begin{aligned} \alpha' m_L^2 &= \frac{\alpha'}{2} \left(\frac{M}{R} + \frac{LR}{\alpha'} \right)^2 + 2(N_L - 1), \\ \alpha' m_R^2 &= \frac{\alpha'}{2} \left(\frac{M}{R} - \frac{LR}{\alpha'} \right)^2 + 2(N_R - 1), \\ \alpha' m^2 &= \alpha'(m_L^2 + m_R^2) = \alpha' \frac{M^2}{R^2} - \frac{1}{\alpha'} L^2 R^2 + 2(N_L + N_R - 2), \end{aligned} \quad (2.2.4)$$

where $\frac{M^2}{R^2}$ comes from the momentum in the compact dimension, while the term $\frac{1}{\alpha'} L^2 R^2$ corresponds to the energy required to wrap the string around the circle L times. Physical states have to satisfy the reparametrization constraint

$$m_L^2 = m_R^2. \quad (2.2.5)$$

The T-duality transformation inverts the radius of the circle $R \rightarrow \tilde{R} = \alpha'/R$ and leaves the mass formula of the string invariant providing that the string winding states are exchanged with the Kaluza-Klein modes.

2.2.2 T-duality of superstrings

Superstring theory incorporates Majorana worldsheet fermions as the superpartners of all worldsheet bosons. We denote the fermionic fields as ψ_L^i for the left-movers and ψ_R^i as the right-movers. The resulting theory exhibits $N = (1, 1)$ supersymmetry on the worldsheet. The boundary conditions and T-duality for the bosonic closed sector are the same as we discussed in 2.2.1. In addition, there are new boundary conditions with regards to the fermionic degrees of freedom,

$$\psi_{L/R}^i(\tau, 0) = \pm \psi_{L/R}^i(\tau, 2\pi). \quad (2.2.6)$$

This gives rise to four different sectors, NS/NS, NS/R, R/NS, R/R when NS corresponds to the sector with Neveu-Schwarz boundary conditions and R stands for Ramond boundary

conditions. In addition, there is a Gliozzi–Scherk–Olive (GSO) projection rendering the spectrum tachyon free and spacetime supersymmetry. The action of the right moving GSO projection on the Ramond sector ground state can be expressed in light cone gauge as

$$(-1)^F = 16 \Pi_{i=2}^9 b_0^i, \quad (2.2.7)$$

with b_0^i being the zero-modes of ψ_R^i and F is the number of the worldsheet fermions. The T-duality transformation along a single S^1 circle changes the sign of the right-moving GSO projection in the Ramond sector, for type IIB we have $(-1)^F = (-1)^{\bar{F}} = 1$, while for type IIA we have $(-1)^F = -(-1)^{\bar{F}} = 1$. We can see that if we change the sign of the right-moving GSO projection, we exchange the type IIB and type IIA superstring theories. In summary, due to the worldsheet supersymmetry, T-duality also acts on the worldsheet fermions as an asymmetric reflection

$$(\phi_L, \phi_R) \rightarrow (\phi_L, -\phi_R). \quad (2.2.8)$$

For an odd number of T-duality transformations, IIB is dual to IIA superstring theory, while for an even number, they map to themselves.

Imposing the supersymmetric extension to the right-movers while keeping the left-movers bosonic, one obtains a hybrid construction of heterotic superstring theories with $N = 1$ target space supersymmetry. Compactification of D bosonic coordinates on a D -dimensional torus T^D , the resulting theory is effectively $(26 - D)$ dimensional. The torus is defined by identifying points in the D -dimensional internal space as

$$x^I \sim x^I + 2\pi \sum_{i=1}^D n^i e_i^I = x^I + 2\pi L^I, \quad n^i \in \mathbb{Z}, \quad (2.2.9)$$

with $L^I = \sum_{i=1}^D n^i e_i^I$, $n^i \in \mathbb{Z}$. In the heterotic superstring construction, we have $D = 16$. This leads to 10 uncompactified bosonic fields $X_L^\mu(\tau + \sigma)$ with $\mu = 0 \dots 9$ and 16 internal bosons $X_L^I(\tau + \sigma)$ with $\mu = 1 \dots 16$ which live on a 16-dimensional torus. The right movers contains 10 uncompactified bosonic fields $x_R^\mu(\tau - \sigma)$ and their fermionic superpartners $\psi_R^\mu(\tau - R)$ with $\mu = 0 \dots 9$. The momentum of the additional chiral bosons ($x_L^I(\tau + \sigma)$) are discrete, as the vectors of a 16-dimensional lattice Γ^{16} ,

$$\mathbf{p}_L \in \Gamma_{16}, \quad p_L^I = p_i \quad e_i^I, \quad I = 1 \dots 16, \quad p_i \in \mathbb{Z}. \quad (2.2.10)$$

The e_i^I are the basis vectors of Γ_{16} , and its metric is

$$g_{ij} = \sum_{I=1}^{16} e_i^I e_{jI}. \quad (2.2.11)$$

By studying the partition function, modular invariance of the one loop partitions implies that the internal 16-dimensional momentum lattice Γ_{16} must be an even self dual Euclidean lattice. These lattices are very rare. In 16-dimension, there are only two even self-dual Euclidean lattices: the direct product lattice $\Gamma_{E8} \times \Gamma_{E8}$, where $E8$ is the root lattice of $E8$, and Γ_{D16} which is the weight lattice of $Spin(32)/\mathbb{Z}_2$ (contains the root lattice of $SO(32)$). These correspond to $E8 \times E8$ and $SO(32)$ superstring respectively. The $E8 \times E8$ and $SO(32)$ superstring theories have the same number of states at every mass level which are organized under different internal gauge symmetries. They share the same partition function but with different correlation functions.

We know that if we consider the compactification of the heterotic $E8 \times E8$ on an S^1 of radius R , in the absence of a gauge background it is invariant under the T-duality transformation $R \rightarrow \alpha'/R$ and exchange of momentum and winding numbers. The same is true for heterotic $SO(32)$ superstring. If we choose in each theory an appropriate gauge background, breaking the gauge symmetry to $SO(16) \times SO(16)$ and relate the radii of the two circles as $R_1 R_2 = \alpha'/2$. One can show that the two theories have identical spectra and symmetries. This is the T-duality between heterotic $E8 \times E8$ and $SO(32)$ superstring theories.

For compactification on higher dimensional tori, we can generalize the T-dualities to mirror symmetry, which is defined according to compactifications on Calabi-Yau manifolds. It can be interpreted as performing a sequence of T-dualities. Each Calabi-Yau manifold CY is associated with a mirror Calabi-Yau manifold CY' by interchange the Kähler modulus and complex structure. Compactifications of type IIA and IIB superstring theories on mirror-dual Calabi-Yau manifolds give rise to dual versions of effective theories.

2.2.3 S-duality

Besides T-duality, another most important duality in superstring theories is S-duality. S-duality identify theories with strong and weak coupling region. It allows to study the strong coupling theory by studying the weak coupling dual theory. Furthermore it is beyond the perturbative regime (distinct from T-duality). It connects type IIA and heterotic $E8 \times E8$ superstring theories with eleven dimensional **M-theory**. Thus all five superstring theories are unified into a 11-dimensional M-theory. This also includes the existence D-branes, coming from higher-dimensional objects named M-branes. The low energy effective theory of M-theory is known to be 11-dimensional supergravity. The unification version of S-duality and T-duality is named U-duality. The various dualities indicate that distinct ten-dimensional superstring theories are considered as different limits of a more fundamental theory. The unification of 5 types of superstring theory are shown in Figure 2.3. This strong/weak duality is actually part of a larger symmetry group $SL(2, \mathbb{Z})$. This symmetry group admits a geometric interpretation in terms of two additional toroidal dimensions. This raises up a new twelve dimensional construction manifesting $SL(2, \mathbb{Z})$ named **F-theory**. The additional two dimensions are necessarily a compact torus, which

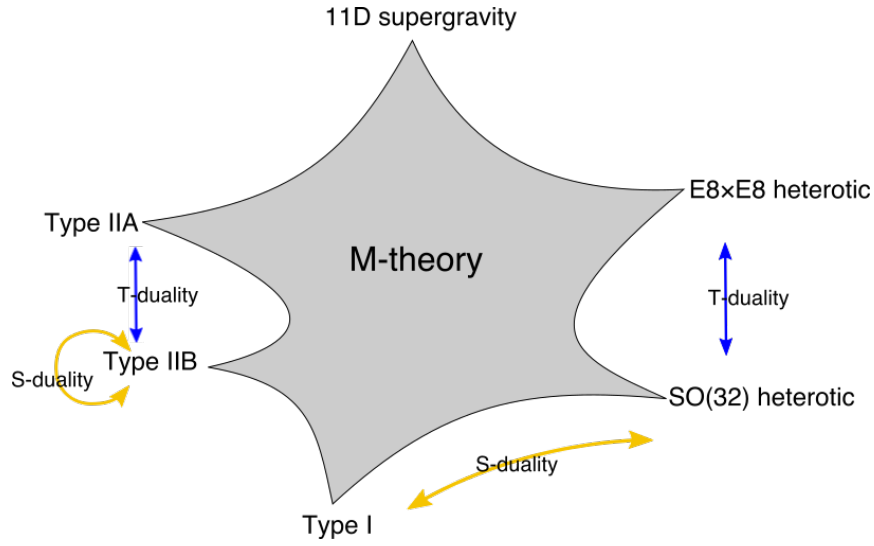


Figure 2.3: Illustration of theories unified in M-theory

non-trivially fibered over the compactification manifold. On the other hand, type IIB superstring theory is the S-dual of itself. F-theory can be considered as a generalization of type IIB superstring theory with S-duality manifested via $SL(2, \mathbb{Z})$ symmetry.

2.3 Double field theory

In this section, we summarize the main features of the DFT formulation, as it has been described in [63, 66], based on the earlier work [78, 79] and [100, 122, 123]. For a more concrete introduction we refer to these papers. As we discussed in the introduction, DFT is a proposal to incorporate T-duality, as a symmetry of a field theory defined on a double configuration space. This features a global symmetry group $O(D, D)$. We first review the basic ideas on T-duality and supergravity in order to construct the generalized diffeomorphisms and an invariant action on the double space.

Recall that T-duality in string theory is suggested by compactification of strings on a torus. In string toroidal compactifications, there are compact momentum modes corresponding to the compact coordinates y^m , as well as T-dual winding modes. If we describe the winding modes by introducing a new kind of coordinates, winding coordinates \tilde{y}_m , then the description of the winding modes could be similar as the momentum modes. Formally we also assign a dual winding coordinate \tilde{x}_μ to the non-compact spacetime coordinate x^μ , thus the normal coordinates are all doubled, and the fields depend on generalized coordinates $X^M = (\tilde{x}_\mu, \tilde{y}_m, x^\mu, y^m)$. Moreover, when the compactification scale is much bigger than the string scale, it is hard for the string to warp and thus no winding modes appear. This corresponds to the usual case in DFT when a so-called strong constraint is implemented. On the contrary, when the compactification scale is small, the winding modes become dominant and DFT only depends on dual winding coordinates. Based on these intuitions, we introduce the framework of DFT in the next sections.

2.3.1 Review of double field theory

DFT is constructed as a T-duality invariant formulation of the low-energy effective description of string theory. The T-duality symmetry of the circle compactification is generalized into $O(D, D)$ in the toroidal compactification with constant background metric and antisymmetric B-field. The main new feature of DFT is that one doubles the number of coordinates by introducing winding coordinates \tilde{x}_m and arranges them into a doubled vector $X^M = (\tilde{x}_m, x^m)$. An $O(D, D)$ action is defined that preserve the $O(D, D)$ invariant metric η_{MN} ,

$$h_M{}^P \eta_{PQ} h_N{}^Q = \eta_{MN} \quad (2.3.1)$$

where

$$\eta_{MN} = \begin{pmatrix} 0 & \delta_m{}^n \\ \delta_m{}^n & 0 \end{pmatrix}, \quad \eta^{MN} = \begin{pmatrix} 0 & \delta_m{}^n \\ \delta_m{}^n & 0 \end{pmatrix}, \quad \eta^{MP} \eta_{PN} = \delta_N^M, \quad (2.3.2)$$

with an $O(D, D)$ generalized frame

$$\eta_{MN} = E^A{}_M \eta_{AB} E^B{}_N, \quad (2.3.3)$$

where η_{AB} raises and lowers flat indices and takes the same form as η_{MN} (2.3.2). A generalized vielbein $E^A{}_M$ with metric is introduced as follows

$$S_{AB} = \begin{pmatrix} s^{ab} & 0 \\ 0 & s_{ab} \end{pmatrix} \quad (2.3.4)$$

with $s_{ab} = \text{diag}(- + \dots +)$ being the flat D -dimensional Minkowski metric. The parametrization of this generalized vielbein reads

$$E^A{}_M = \begin{pmatrix} e_a{}^m & e_a{}^k B_{km} \\ 0 & e^a{}_m \end{pmatrix}, \quad (2.3.5)$$

with the ordinary vielbein $e_a{}^m s^{ab} e_b{}^n = g^{mn}$. These can be unified into a asymmetric generalized metric \mathcal{H}_{MN} given by

$$\mathcal{H}_{MN} = E^A{}_M S_{AB} E^B{}_N, \quad (2.3.6)$$

and expressed as

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} B_{kj} \\ B_{ik} g^{kj} & g_{ij} - B_{ik} g^{kl} B_{lj} \end{pmatrix}. \quad (2.3.7)$$

This is an $O(D, D)$ element and its inverse is obtained by raising the indices with the $O(D, D)$ metric η^{MP} introduced in (2.3.2)

$$\mathcal{H} \in O(D, D), \quad \mathcal{H}^{MN} = \eta^{MP} \mathcal{H}_{PQ} \eta^{QP}, \quad \mathcal{H}_{MP} \mathcal{H}^{PN} = \delta_M^N. \quad (2.3.8)$$

All the indices in DFT are raised and lowered with the $O(D, D)$ invariant metric (2.3.2). The dilaton ϕ is combined with the determinant of the metric g into an $O(D, D)$ scalar d

$$e^{-2d} = \sqrt{g} e^{-2\phi}. \quad (2.3.9)$$

Constraints

DFT is required to be invariant under a large symmetry group. At first it is invariant under global $G = O(D, D)$ transformations and secondly it is invariant under a local $H \subset G$ symmetry with $H = O(D) \times O(D)$. The generalized metrics \mathcal{H}_{MN} are parametrized in the coset G/H . This local symmetry acts on the vielbein as

$$\delta_\Lambda E_A{}^M = \Lambda_A{}^B E_B{}^M \quad \text{with} \quad \Lambda_A{}^C S_{CD} \Lambda_B{}^D = S_{AB}, \quad (2.3.10)$$

so that they can be viewed as local double Lorentz transformations. Besides that, the usual diffeomorphism symmetry is enhanced to so-called generalized diffeomorphisms with infinitesimal parameter $\xi^M = (\tilde{\lambda}_m, \lambda^m)$ and generalized Lie-derivative, acting e.g. on a doubled vector V as

$$\mathcal{L}_\xi V^M = \xi^N \partial_N V^M + (\partial^M \xi_N - \partial_N \xi^M) V^N. \quad (2.3.11)$$

For instance the vielbeins E_A transform as a doubled vector, whereas the dilaton d transforms as a scalar density

$$\delta_\xi d = \mathcal{L}_\xi d = \xi^M \partial_M d - \frac{1}{2} \partial_M \xi^M. \quad (2.3.12)$$

This allows to define a generalized tensor calculus by defining that the variation of a tensor with respect to generalized diffeomorphisms is

$$\delta_\xi T^{M_1 \dots M_k} = \mathcal{L}_\xi T^{M_1 \dots M_k}. \quad (2.3.13)$$

In contrast to the usual Lie-derivative, the Lie-derivative of a generalized tensor is not automatically again a generalized tensor. To ensure this, one has to impose the so-called *closure constraint*

$$\Delta_{\xi_1} (\mathcal{L}_{\xi_2} T^{M_1 \dots M_k}) = 0 \quad (2.3.14)$$

with the anomalous variation $\Delta(\cdot) = \delta_\xi(\cdot) - \mathcal{L}_\xi(\cdot)$. This suggests that the symmetry algebra closes [100], i.e. that a Lie-derivative of a generalized tensor is again a generalized tensor (2.3.14). Scherk-Schwarz reductions are prototype examples, whose reduced action is closely related to gauged supergravity and whose internal spaces are truly non-geometric in the sense that fields depend on doubled coordinates (y^m, \tilde{y}_m) .

Apart from the closure constraint, there are also the so-called *weak* and *strong constraints*

$$\partial_M \partial^M = 0, \quad \partial_M f \partial^M g = \mathcal{D}_A f \mathcal{D}^A g = 0, \quad (2.3.15)$$

with f, g being the fundamental objects like E^A_M and ξ^M . Locally, up to an $O(D, D)$ transformation these constraints remove the winding dependence by choosing $\tilde{\partial}^i = 0$. In particular, the constraints guarantee the closure constraint. In the following, we always implement the weak and strong constraint for the uncompactified directions.

Action

Recall that DFT is originally constructed as a generalization of supergravity incorporating T-dualities in the version of global $O(D, D)$ symmetry. Here we show details. The bosonic

NS-NS sector of the supergravity action takes the form of

$$\mathcal{S} = \int dx \sqrt{g} e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} \right), \quad (2.3.16)$$

where the three-form H_{ijk} and the corresponding Bianchi identity take the forms

$$H_{ijk} = 3\partial_{[i} B_{jk]}, \quad \partial_{[i} H_{ijk]} = 0, \quad (2.3.17)$$

and R is the Ricci scalar.

Imposing the strong constraint $\tilde{\partial}^i = 0$ (which annihilates the winding coordinates dependence) and inserting all the elements of generalized metric (2.3.7), we can reproduce (2.3.16) by writing the generalized action in DFT as

$$\begin{aligned} S = \int dX e^{-2d} & \left(4\mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\mathcal{H}^{MN} \partial_M d \partial_N d + 4\partial_M \mathcal{H}^{MN} \partial_N d \right. \\ & \left. + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_K \mathcal{H}_{NL} + \Delta_{(SC)} \mathcal{R} \right), \end{aligned} \quad (2.3.18)$$

in which we denote all the terms that vanish under strong constraint as $\Delta_{(SC)} \mathcal{R}$.

Flux Formulations

A more generic parameterization of the generalized vielbein (compare to (2.3.5)), including both a two form B_{mn} and a two-vector β^{mn} reads

$$E^A_M = \begin{pmatrix} e_a^m & e_a^k B_{km} \\ e^a_k \beta^{km} & e^a_m + e^a_k \beta^{kl} B_{lm} \end{pmatrix}. \quad (2.3.19)$$

When $\beta^{mn} = 0$, we call it in standard geometric frame, while when $B_{mn} = 0$ it is in a so-called non-geometric frame. The flat derivative is defined as

$$\mathcal{D}^A = E^A_M \partial^M. \quad (2.3.20)$$

Using these beins, one defines the generalized fluxes \mathcal{F}_{ABC} as

$$\mathcal{F}_{ABC} = 3\Omega_{[ABC]}, \quad (2.3.21)$$

in terms of the generalized Weitzenböck connection¹

$$\Omega_{ABC} = \mathcal{D}_A E_B^M E_{CM}. \quad (2.3.22)$$

¹For a recent discussion of the role of a Weitzenböck connection in DFT, see [124].

The components of these DFT fluxes \mathcal{F}_{ABC} are precisely the geometric and non-geometric fluxes H, F, Q and R

$$\mathcal{F}_{abc} = H_{abc}, \quad \mathcal{F}^a{}_{bc} = F^a{}_{bc}, \quad \mathcal{F}_c{}^{ab} = Q_c{}^{ab}, \quad \mathcal{F}^{abc} = R^{abc}. \quad (2.3.23)$$

The T-duality invariant dilaton transform as

$$e^{-2d} = e^{-2\phi} \sqrt{g} \quad (2.3.24)$$

which is also used to define the flux

$$\mathcal{F}_A = \Omega^B{}_{BA} + 2E_A{}^M \partial_M d. \quad (2.3.25)$$

The invariant action of the flux formulation of DFT reads

$$\begin{aligned} S_{\text{DFT}} = \int dX \, e^{-2d} & \left[\mathcal{F}_A \mathcal{F}_{A'} S^{AA'} + \mathcal{F}_{ABC} \mathcal{F}_{A'B'C'} \left(\frac{1}{4} S^{AA'} \eta^{BB'} \eta^{CC'} - \frac{1}{12} S^{AA'} S^{BB'} S^{CC'} \right) \right. \\ & \left. - \frac{1}{6} \mathcal{F}_{ABC} \mathcal{F}^{ABC} - \mathcal{F}_A \mathcal{F}^A \right]. \end{aligned} \quad (2.3.26)$$

Note that in CFT we can assign a worldsheet parity Ω to every field (see e.g. [67]). Then, the terms in the first line are Ω -even and the terms in the second line are Ω -odd. The flux formulation of DFT is an extension of the generalized metric formulation. The action (2.3.26) can be recast in the form of (2.3.18) up to total derivatives.

2.3.2 Heterotic double field theory

In this section we briefly review the bosonic sector of heterotic DFT, where we focus on those features which are important for our later discussion in chapter 4. The bosonic NS-NS sector of heterotic DFT with abelian gauge fields is a straightforward generalization of bosonic DFT as expected [94]. The abelian gauge fields appear by dimensional reduction of heterotic superstring theory. The low-energy effective action of the massless bosonic sector for the heterotic string is described by the action

$$\mathcal{S} = \int dx \sqrt{g} e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} - \frac{1}{4} G^{ij}{}_{\alpha} G_{ij}{}^{\alpha} \right), \quad (2.3.27)$$

in which the field strength of the non-abelian gauge fields is defined as

$$G_{ij}{}^{\alpha} = \partial_i A_j{}^{\alpha} - \partial_j A_i{}^{\alpha} + g_0 [A_i, A_j]{}^{\alpha}, \quad (2.3.28)$$

and the strength of the Kalb-Ramond field is modified by the Chern-Simons three-form,

$$H_{ijk} = 3 \left(\partial_{[i} B_{jk]} - \kappa_{\alpha\beta} A_{[i}^{\alpha} \partial_j A_{k]}^{\beta} - \frac{1}{3} g_0 \kappa_{\alpha\beta} A_{[i}^{\alpha} [A_j, A_k]^{\beta} \right). \quad (2.3.29)$$

Here $\kappa_{\alpha\beta}$ denotes the Cartan-Killing metric of the gauge group. In the abelian case, this is simply the unit matrix, $\kappa_{\alpha\beta} = \delta_{\alpha\beta}$. Note that the order in α' can be made visible by scaling $A_i^{\alpha} \rightarrow \sqrt{\alpha'} A_i^{\alpha}$. In the DFT formulation of the abelian heterotic string [94], for each gauge field A^{α} one introduces a new coordinate y^{α} so that the entire DFT lives on a $2D + n$ dimensional space with coordinates

$$X^M = (\tilde{x}_i, x^i, y^{\alpha}). \quad (2.3.30)$$

The global symmetry group is enhanced from $O(D, D)$ to $O(D, D+n)$ as the generalization of the T-duality group for heterotic superstring theory. The doubled coordinates X^M transform as an $O(D, D+n)$ vector

$$X'^M = h^M_N X^N, \quad h \in O(D, D+n). \quad (2.3.31)$$

As in bosonic DFT, one introduces an $O(D, D+n)$ invariant metric

$$\eta_{MN} = \begin{pmatrix} 0 & \delta_j^i & 0 \\ \delta_i^j & 0 & 0 \\ 0 & 0 & \delta_{\alpha\beta} \end{pmatrix} \quad (2.3.32)$$

satisfying

$$\eta^{MN} = h^M_P h^N_Q \eta^{PQ}. \quad (2.3.33)$$

This $O(D, D+n)$ metric is used to raise and lower capital indices like M . Accordingly, the generalized derivatives and gauge parameters take the form of

$$\partial_M = (\tilde{\partial}^i, \partial_i, \partial_{\alpha}), \quad \xi^M = (\tilde{\xi}_i, \xi^i, \Lambda^{\alpha}). \quad (2.3.34)$$

As it is shown in [94], one can introduce a generalized Lie derivative and a C-bracket. The closure of the algebra is guaranteed, if one introduces the strong constraint in heterotic DFT,

$$\partial_M f \partial^M g = \tilde{\partial}^i f \partial_i g + \partial_i f \tilde{\partial}^i g + \partial_{\alpha} f \partial^{\alpha} g = 0, \quad (2.3.35)$$

where f and g are arbitrary fields and gauge parameters. This means that the heterotic level-matching condition

$$\partial_M \partial^M f = 2 \tilde{\partial}^i \partial_i f + \partial_{\alpha} \partial^{\alpha} f = 0 \quad (2.3.36)$$

also has to hold for products of fields. This implies that locally there exists an $O(D, D+n)$ transformation rotating the coordinates into a frame in which the fields only depend on the normal coordinates x^i .

The generalized metric

The heterotic DFT action can be expressed in terms of a generalized metric and an $O(D, D+n)$ invariant dilaton d defined by $e^{-2d} = \sqrt{g}e^{-2\phi}$. The metric \mathcal{H}^{MN} transforms covariantly under $O(D, D+n)$

$$\mathcal{H}'^{MN}(X') = h^M{}_P h^N{}_Q \mathcal{H}^{PQ}(X) \quad (2.3.37)$$

and is parameterized in terms of the metric g_{ij} , the Kalb-Ramond field B_{ij} and the gauge fields A_i^α as

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}C_{kj} & -g^{ik}A_{k\beta} \\ -g^{jk}C_{ki} & g_{ij} + C_{ki}g^{kl}C_{lj} + A_i^\gamma A_{j\gamma} & C_{ki}g^{kl}A_{l\beta} + A_{i\beta} \\ -g^{jk}A_{k\alpha} & C_{kj}g^{kl}A_{l\alpha} + A_{j\alpha} & \delta_{\alpha\beta} + A_{k\alpha}g^{kl}A_{l\beta} \end{pmatrix} \quad (2.3.38)$$

where C_{ij} takes the form

$$C_{ij} = B_{ij} + \frac{1}{2}A_i^\alpha A_{j\alpha}, \quad (2.3.39)$$

and splits into a symmetric part and an antisymmetric two-form

$$C_{(ij)} = \frac{1}{2}A_i^\alpha A_{j\alpha}, \quad C_{[ij]} = B_{ij}. \quad (2.3.40)$$

Implementing the heterotic strong constraint $\tilde{\partial}^i = \partial_\alpha = 0$ which annihilates the winding and gauge coordinates dependence, the heterotic DFT action (parametrized by the generalized metric (2.3.38)) is identical to the massless bosonic sector of heterotic string (2.3.27),

$$S = \int dx e^{-2d} \left(\frac{1}{8} \mathcal{H}^{ij} \partial_i \mathcal{H}^{KL} \partial_j \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{Mi} \partial_i \mathcal{H}_{Kj} \partial_j \mathcal{H}_{MK} - 2 \partial_i d \partial_j \mathcal{H}^{ij} + 4 \mathcal{H}^{ij} \partial_i d \partial_j d \right). \quad (2.3.41)$$

Flux Formulation of heterotic DFT

In analogy to bosonic DFT, one can also introduce a generalized vielbein $E^A{}_M$ so that

$$\mathcal{H}_{MN} = E^A{}_M S_{AB} E^B{}_N \quad (2.3.42)$$

with the constant generalized metric

$$S_{AB} = \begin{pmatrix} s^{ab} & 0 & 0 \\ 0 & s_{ab} & 0 \\ 0 & 0 & s_{\alpha\beta} \end{pmatrix} \quad (2.3.43)$$

where $s_{ab} = \text{diag}(-, +, \dots, +)$, and $s_{\alpha\beta} = \text{diag}(+, \dots, +)$. The generalized vielbein reads²

$$E^A_M = \begin{pmatrix} e_a^i & -e_a^k C_{ki} & -e_a^k A_{k\beta} \\ 0 & e_a^i & 0 \\ 0 & A_i^\alpha & \delta^\alpha_\beta \end{pmatrix}, \quad E_A^M = \begin{pmatrix} e_a^i & 0 & 0 \\ -e_a^k C_{ki} & e_a^i & -e_a^k A_k^\beta \\ A_{i\alpha} & 0 & \delta_\alpha^\beta \end{pmatrix} \quad (2.3.44)$$

which satisfy

$$\eta_{MN} = E^A_M E_{AN}. \quad (2.3.45)$$

The generalized derivative reads

$$D_A = E_A^M D_M = (\tilde{D}^a, D_a, D_\alpha) \quad (2.3.46)$$

with each component taking the form of

$$\begin{aligned} \tilde{D}^a &= \tilde{\partial}^a, \\ D_a &= \partial_a - B_{ai} \tilde{\partial}^i - \frac{1}{2} A_a^\alpha A_{i\alpha} \tilde{\partial}^i - A_a^\gamma \partial_\gamma, \\ D_\alpha &= \partial_\alpha + A_{i\alpha} \tilde{\partial}^i. \end{aligned} \quad (2.3.47)$$

The generalized Weitzenböck connection is defined as

$$\Omega_{ABC} = D_A E_B^N E_{CN}. \quad (2.3.48)$$

The generalized fluxes of heterotic DFT are defined as

$$\mathcal{F}_{ABC} = E_{CM} \mathcal{L}_{E_A} E_B^M = \Omega_{ABC} + \Omega_{CAB} - \Omega_{BAC}. \quad (2.3.49)$$

In a holonomic basis, one finds that the three-form flux takes the form

$$H_{ijk} = -3 \left(\partial_{[i} B_{jk]} - \delta_{\alpha\beta} A_{[i}^\alpha \partial_{\underline{j}} A_{\underline{k}]}^\beta \right), \quad (2.3.50)$$

which is precisely the field strength of the Kalb-Ramond field modified by the Chern-Simons three-form (2.3.29). In section 4.1.2, we will evaluate the generalized fluxes more

²Note that compared to [66], we have two different signs in the definition of the vielbein. This is because we want to be consistent with the generalized metric as defined in [94].

explicitly, and study the global $O(D, D + n)$ transformations acting on these fields.

2.4 Exceptional field theory

Exceptional Field Theory (EFT) is constructed with the motivation to construct a field theory manifest U-duality as the global symmetry group, as an extension of DFT. In many cases, this is manifested into an exceptional group and EFT is considered as a generalization of the effective action of M-theory.

As it was mentioned in the last section, in DFT one introduces an $O(D, D)$ covariant object, generalized metric \mathcal{H}_{MN} , which is parametrized in the coset $O(D, D)/(O(D) \times O(D))$ and written in terms of the D -dimensional metric g_{mn} and the B-field. The usual spacetime coordinates x^a are associated to the normal fields while the dual coordinates x_a are associated to the dual fields in the non-geometric frame. Mathematically, this is realized in generalized geometry by introducing a generalized tangent bundle that is a direct sum of the tangent and cotangent bundles of the spacetime $TM \oplus T^*M$, with only normal coordinates x^a . The base of the bundles is still a D -dimensional space. The standard fields are described in the tangent bundle while the dual fields are described in the cotangent bundle. In the generalization of this formulation to M-theory, the U-duality group is realized via E_d for duality acting in d -dimensions, namely the global symmetry group is E_d and local symmetry group is the maximal compact subgroup H_d . Thus, the generalized metric that unifies the metric g_{mn} and the gauge fields are parametrized in the coset E_d/H_d . Since the fundamental objects of M-theory are represented by M_2 and M_5 branes, the corresponding extended space gives rise to the standard coordinates x^a , dual coordinates y_{ab} for the $M2$ brane, and z_{abcde} for the $M5$ brane etc. The generalized tangent bundle takes the extended form

$$TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus \dots \quad (2.4.1)$$

From the supergravity point of view, the low energy effective theory of M-theory, namely 11-dimensional supergravity, is given by [125]. The action in the bosonic sector takes the form of

$$\mathcal{S}_{11} = \int dx \sqrt{g} \left(R - \frac{1}{48} F_4^2 \right) - \frac{1}{6} F_4 \wedge F_4 \wedge C_3, \quad (2.4.2)$$

where the field strength is defined as $F_4 = dC_3$. It is discovered when one compactifies 11-dimensional supergravity on tori of various dimensions, that exhibits a number of hidden symmetries [64, 126]. More specifically, compactifications on a torus of dimension n results in a $D = 11 - n$ dimensional theory whose global symmetry group is G and maximal local symmetry group is H , as listed in Table 2.4. Here in the same table, we also include the corresponding generalized tangent bundles, starting from $TM \oplus \Lambda^2 T^*M$, which corresponds to the $SL(5)$ theory we will study later.

D	n	G	H	E
10	1	$SO(1, 1)$	1	
9	2	$SL(2)$	$SO(2)$	
8	3	$SL(3) \times SL(2)$	$SO(3) \times SO(2)$	$TM \oplus \Lambda^2 T^* M$
7	4	$SL(5)$	$SO(5)$	$TM \oplus \Lambda^2 T^* M$
6	5	$SO(5, 5)$	$SO(5) \times SO(5)$	$TM \oplus \Lambda^2 T^* M \oplus \Lambda^5 T^* M$
5	6	E_6	$USp(8)$	$TM \oplus \Lambda^2 T^* M \oplus \Lambda^5 T^* M$
4	7	E_7	$SU(8)$	$TM \oplus \Lambda^2 T^* M \oplus \Lambda^5 T^* M \oplus \Lambda^6 TM$
3	8	E_8	$SO(16)$	

Table 2.1: The global symmetry groups G and their maximal compact subgroups H of 11-dimensional supergravity compactified on a torus T^n .

2.4.1 $SL(5)$ exceptional field theory

In the simplest case, from the 11-dimensional supergravity point of view, we review the $SL(5)$ theory with the scalar degrees of freedom appearing in the compactification to seven dimensions. The $SL(5)$ theory is defined on a 10-dimensional extended space. The coordinates x^A lie in the antisymmetric **10**-dimensional representation of $SL(5)$. We write the 10-dimensional index A as an antisymmetric pair of indices in the fundamental **5**-dimensional representation of $SL(5)$, $A \equiv [aa']$, $a, a' = 1, \dots, 5$ [127].

The action

The bosonic fields of the theory are parametrized in the coset of $R^+ \times SL(5)/SO(5)$, which in principle depend on the full ten-dimensional extended coordinates x^{ab} . These fields can be unified into a generalized metric M_{AB} , which parametrizes the given coset and serves as the metric on the extended spacetime. This coset condition allows this generalized metric M_{AB} to be decomposed in terms of a “little metric” m_{ab} with

$$M_{AB} \equiv M_{aa',bb'} = m_{ab}m_{a'b'} - m_{ab'}m_{a'b} \quad (2.4.3)$$

where m_{ab} is a symmetric tensor of rank 2 under $SL(5)$ U-dualities. Although m_{ab} is referred as the little metric, it itself is not a metric on some space. However it contains exactly the right number of degrees of freedom to parametrize the coset $R^+ \times SL(5)/SO(5)$, so it provides a convenient way to construct the theory³. The extra R^+ factor is a consequence of our truncation, and leads to an extra scalar degree of freedom related to the warping of the external seven directions.

The little metric m_{ab} is parametrized with geometric background fields in terms of the

³Note that one can only decompose the full generalized metric in this way in the $SL(5)$ theory, and not for the higher exceptional groups.

usual metric and a three-form, as well as the non-geometric background fields in terms of an alternative metric and a dual trivector. This provides the convenience to study the duality transformation between the geometric and non-geometric frames for $SL(5)$ theory. This little metric is in the form of

$$m_{ab} = e^{-\phi/2} \begin{pmatrix} |g|^{-1/2}(g_{ij} + W_i V_j + V_i W_j + W_i W_j(1 + V^2)) & V_i + W_i(1 + V^2) \\ V_j + W_j(1 + V^2) & |g|^{1/2}(1 + V^2) \end{pmatrix}. \quad (2.4.4)$$

The corresponding $SL(5)$ generalized vielbein is

$$E^\alpha_a = e^{-\phi/4} \begin{pmatrix} e^{-1/2}(e^\mu_i + V^\mu W_i) & e^{1/2}V^\mu \\ e^{-1/2}W_i & e^{-1/2} \end{pmatrix}, \quad (2.4.5)$$

with its inverse

$$E_\alpha^a = e^{\phi/4} \begin{pmatrix} e^{1/2}e_\mu^i & -e^{-1/2}W_\mu \\ -e^{1/2}V^i & e^{-1/2}(1 + V^j W_j) \end{pmatrix}. \quad (2.4.6)$$

The scalar ϕ comes from the truncation, which takes $e^\phi = |g_7|^{1/7}$, where g_7 is the determinant of the metric in the external directions. The vector V^i is a dualization of the three-form,

$$V^i = \frac{1}{3!} \varepsilon^{ijkl} C_{jkl}, \quad (2.4.7)$$

and the covector W_i is the dualization of an antisymmetric field,

$$W_i = \frac{1}{3!} \varepsilon_{ijkl} \Omega^{jkl}. \quad (2.4.8)$$

In the geometric frame, we set $W_i = 0$, hence the generalized little metric (2.4.4) becomes

$$m_{ab} = e^{-\phi/2} \begin{pmatrix} |g|^{-1/2}g_{ij} & V_i \\ V_j & |g|^{1/2}(1 + V^2) \end{pmatrix}. \quad (2.4.9)$$

In the non-geometric frame, we set $V^i = 0$ instead, and the generalized metric becomes

$$\tilde{m}_{ab} = e^{-\tilde{\phi}/2} \begin{pmatrix} |\tilde{g}|^{-1/2}(\tilde{g}_{ij} + W_i W_j) & W_i \\ W_j & |\tilde{g}|^{1/2} \end{pmatrix}. \quad (2.4.10)$$

By applying a $SL(5)$ transformation to the generalized metric in the geometric frame, and with a proper field redefinition, one can arrive in the non-geometric frame which we will show in chapter 4.

3

String Compactification

3.1 Type IIB string compactification

In this section, we introduce the low energy effective action of Calabi-Yau orientifold compactifications of type IIB superstring theory. By doing Calabi-Yau compactifications, e.g. the internal space is Calabi-Yau threefolds, one obtains an $N = 2$ theory in four-dimensions. Recall that string theory is not only a theory of fundamental strings but also contains higher-dimensional extended objects such as D-branes and orientifold planes. D-branes have massless excitations with attached open strings while orientifold planes carry no physical degrees of freedom. By introducing appropriate D-branes and/or orientifold planes, the $N = 2$ supersymmetry can be further broken to $N = 1$ effective theory. Turning on background fluxes can spontaneously break the remaining $N = 1$ supersymmetry. We first give an introduction to the basics of compactifications and Calabi-Yau manifolds, then we consider flux compactifications and thus study the phenomenological effects of the theory.

3.1.1 Type IIB effective action

The ten-dimensional supergravity theory is considered as the low energy limit of type II theories at tree level. The field contents encoded in this theory are massless. In the NS-NS sector, it incorporates the metric tensor g_{MN} , the antisymmetric B-field B_{MN} , and the dilaton ϕ . In addition, we have RR p -forms. Depending on which type II theories we consider, p can be either odd (type IIA) or even (type IIB). Here we consider type IIB superstring theory, in which the massless fermions are included. However, we focus only on the bosonic ingredients where we have $p = 0, 2, 4$ in the R-R sector. This includes an antisymmetric self-dual 4-form C_4 , a 2-form C_2 and a 0-form axion C_0 . The other p -forms C_8 and C_6 are dual to C_0 and C_2 , respectively. C_4 is dual to itself. The field strengths \mathfrak{F}_5 , \mathfrak{F}_3 , and \mathfrak{F}_1 are derived from $\mathfrak{F}_{p+1} = dC_p$ respectively. The massless fields are listed in Table 3.1.1. The bosonic part of the ten-dimensional type IIB supergravity effective action

NS-NS sector	R-R sector	Fermions
g_{MN} : metric	C_{MNKL} : self-dual four-form	Φ^i_M : gravitino
B_{MN} : B-field	C_{MN} : two-form	
ϕ : dilaton	C_0 : axion	λ^i : dilatino

Table 3.1: The massless spectrum of Type IIB effective action

is given by

$$\mathcal{S}_{IIB} = \mathcal{S}_{NS} + \mathcal{S}_{RR} + \mathcal{S}_{CS}, \quad (3.1.1)$$

where

$$\begin{aligned} \mathcal{S}_{NS} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\det g_{MN}} e^{-2\phi} [R_{10} + 4\partial^M \phi \partial_M \phi - \frac{1}{2}|H_3|^2], \\ \mathcal{S}_{RR} &= -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-\det g_{MN}} [| \mathfrak{F}_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2], \\ \mathcal{S}_{CS} &= -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge \mathfrak{F}_3, \end{aligned} \quad (3.1.2)$$

with

$$\begin{aligned} H_3 &= dB_2, \quad \mathfrak{F}_{p+1} = dC_p, \quad p = 0, 2, 4, \\ \tilde{F}_3 &= \mathfrak{F}_3 - C_0 H_3, \quad \tilde{F}_5 = \mathfrak{F}_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge \mathfrak{F}_3, \quad \tilde{F}_5 = \star_{10} \tilde{F}_5. \end{aligned} \quad (3.1.3)$$

3.1.2 String compactifications

Type IIB superstring theory lives in a ten-dimensional spacetime \mathcal{M}_{10} . In the compactification we require that \mathcal{M}_{10} can be decomposed into a product of d -dimensional compact space \mathcal{X}_d (internal space) and a $10 - d$ -dimensional spacetime \mathcal{M}_{10-d} (external spacetime):

$$\mathcal{M}_{10} = \mathcal{M}_{10-d} \times \mathcal{X}_d \quad (3.1.4)$$

The decomposition breaks the local lorentz symmetry to:

$$SO(9, 1) \rightarrow SO(9 - d, 1) \times SO(d) \quad (3.1.5)$$

The background metric g_{MN} on \mathcal{M}_{10} is decomposed as

$$ds_{10}^2 = g_{MN} dx^M \otimes dx^N \equiv g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{mn} dy^m \otimes dy^n, \quad (3.1.6)$$

where x^M, x^μ, y^m are the coordinates defined on $\mathcal{M}_{10}, \mathcal{M}_{10-d}, \mathcal{X}_d$, respectively. The background metric on \mathcal{M}_{10-d} is denoted as $g_{\mu\nu}$ while the one on \mathcal{X}_d is g_{mn} . In addition, the internal metric g_{mn} behaves as constant if it is seen from the viewpoint of \mathcal{M}_{10-d} . Similarly the other tensor and spinor fields in the theory can also be decomposed under (3.1.4) accordingly. The geometric structure of \mathcal{M}_{10-d} and \mathcal{X}_d , and the degrees of freedom of the field contents highly depend on the supersymmetry breaking caused by compactification manifold.

In compactifications, a so-called toroidal compactification is one of the simplest examples. The d -dimensional torus T^d is homeomorphic to the Cartesian product of d circles, namely $T^d = S^1 \times S^1 \cdots \times S^1$. This implies that T^d is a flat manifold and a constant spinor defined on T^d preserves supersymmetry which is not favorable from phenomenology. Thus, Calabi-Yau manifolds which better behave in supersymmetry breaking in compactification come into play.

3.2 Calabi-Yau orientifold compactification

3.2.1 Calabi-Yau manifolds

Definitions

A Calabi-Yau n -fold denoted by CY_n is a complex n -dimensional compact Kähler manifold with a holonomy group $H = SU(n)$. There exists one covariantly invariant spinor η for the holonomy group $H = SU(n)$. A Calabi-Yau n -fold is a Ricci-flat Kähler manifold. The metric of a Kähler manifold can be locally written using a Kähler potential, $G_{ij} = \partial_i \bar{\partial}_j K(z, \bar{z})$, which is equivalent to the closeness of the Kähler form. The Kähler potential can not be continuous everywhere in general. In the intersection between two charts U_i and U_j , the Kähler potential transform as:

$$K(z, \bar{z}) \rightarrow K(z, \bar{z}) + f(z) + f(\bar{z}), \quad (3.2.1)$$

which leaves the metric g_{ij} invariant.

In other words, a Calabi-Yau can also be defined by the vanishing of the first Chern class, $c_1(CY_n) = 0$, which is determined by the cohomology class of the Ricci curvature two-form.

Calabi-Yau threefold

A Calabi-Yau threefold has complex dimension 6, with which the 10-dimensional superstring can be exactly compactified to 4-dimensional physics. Thus Calabi-Yau threefolds

draw particular interest in string compactifications. One can construct an $SU(3)$ -invariant differential two-form J_2 and three-form Ω_3 by construction.

They correspond to the Kähler two-form J_2 and the non-vanishing holomorphic three-form Ω_3 respectively. The Kähler form

$$J_2 = \frac{1}{2} J_{mn} dy^m \wedge dy^n \equiv ig_{i\bar{j}} dz^i \wedge d\bar{z}^j, \quad (3.2.2)$$

is closed by definition,

$$0 \equiv dJ_2 = \frac{1}{2!} \partial_{[l} J_{mn]} dy^l \wedge dy^m \wedge dy^n, \quad (3.2.3)$$

where z^i and \bar{z}^j are complex coordinates constructed by rearranging the real coordinates y^m . The nowhere vanishing holomorphic three-form is denoted by Ω_3 ,

$$\Omega_3 = \Omega_{ijk}(z) dz^i \wedge dz^j \wedge dz^k, \quad (3.2.4)$$

which is necessarily closed, namely the decomposition of the exterior derivative

$$\begin{aligned} d\Omega_3 &= (\partial + \bar{\partial}) \Omega_3 \\ &= \partial_i \Omega_{pqr} dz^i \wedge dz^p \wedge dz^q \wedge dz^r + \bar{\partial}_{\bar{i}} \Omega_{pqr}(z) d\bar{z}^{\bar{i}} \wedge dz^p \wedge dz^q \wedge dz^r \\ &= 0. \end{aligned} \quad (3.2.5)$$

3.2.2 Orientifold projections

Under type IIB Calabi-Yau compactifications one approach is the $N = 2$ effective action in four dimensions. Furthermore, one can reduce the supersymmetry by introducing an orientifold projection. Here we review the definition of orientifold projection, orientifold planes and then we study the effects of orientifold projections on cohomology, moduli and compactifications.

Definitions

An orientifold projection is defined as

$$\mathcal{O} = (-1)^{F_L} \Omega_{\sigma^*}, \quad (3.2.6)$$

where Ω is worldsheet parity transformation and σ^* is a reflection in CY_3 . The worldsheet parity is given by

$$\Omega : \sigma \rightarrow 2\pi - \sigma, \quad (3.2.7)$$

where σ is a worldsheet parameter. In type IIB theory, a holomorphic involution σ^{*1} is required to act on the holomorphic three-form Ω_3 and Kähler form J_2 as

$$\sigma^*(\Omega_3) = \pm\Omega_3, \quad \sigma^*(J_2) = J_2. \quad (3.2.8)$$

The operator F_L counts the number of the left-moving spacetime fermions from R-R sector.

Orientifold planes

The holomorphic three-form Ω_3 and Kähler form J_2 in local coordinates are denoted as

$$\Omega_3 \sim dz^1 \wedge dz^2 \wedge dz^3, \quad J_2 \sim \sum_i dz^i \wedge d\bar{z}^{\bar{i}}, \quad (3.2.9)$$

where $z^k = y^{2k+1} + iy^{2k}$, $k = 1, 2, 3$. As it is shown above, there are two choices for the action of σ^* on Ω_3 . With the plus sign, we have

$$\begin{aligned} \sigma^*(J) &= \sum_i \sigma^*(dz^i) \wedge \sigma^*(d\bar{z}^{\bar{i}}), \\ \sigma^*(\Omega) &= \sigma^*(dz^1) \wedge \sigma^*(dz^2) \wedge \sigma^*(dz^3) \\ &\stackrel{!}{=} dz^1 \wedge dz^2 \wedge dz^3. \end{aligned} \quad (3.2.10)$$

We have no fixed coordinates since

$$\sigma^*(y^{2k-1}) = y^{2k-1}, \quad \sigma^*(y^{2i}) = y^{2i}. \quad (3.2.11)$$

This defines $O9$ -planes. If we take the other projection we have instead:

$$\sigma^*(y^{2k-1}) = -y^{2k-1}, \quad \sigma^*(y^{2k}) = -y^{2k}, i = 1, 2. \quad (3.2.12)$$

This projection fixes $\{y^1, y^2, y^3, y^4\}$ and yields an invariant $O5$ -plane in the theory.

On the other hand, the choice with the minus sign allows us to project the $\{y^m\}$ coordinates differently. In more detail, we can take

$$\sigma^*(y^m) = -y^m, \quad m = 1, 2, \dots, 6. \quad (3.2.13)$$

Since all the internal coordinates are fixed, we can define an $O3$ -plane. An $O7$ -plane arises when we take the projection

$$\sigma^*(y^1) = -y^1, \quad \sigma^*(y^2) = -y^2 \quad (3.2.14)$$

¹This sigma is different with the worldsheet parameter in (3.2.7).

while the others are even. Consequently, we can define a set of Op -planes for each projection in type IIB theory:

$$\begin{aligned} i) \sigma^*(\Omega_3) &= \Omega_3, & \sigma^*(J_2) &= J_2 \rightarrow \text{with } O5/O9\text{-planes}, \\ ii) \sigma^*(\Omega_3) &= -\Omega_3, & \sigma^*(J_2) &= J_2 \rightarrow \text{with } O3/O7\text{-planes}. \end{aligned} \quad (3.2.15)$$

Assuming that all the orientifold planes are spacetime filling, the resulting Op -planes can take the following configurations:

$$\begin{aligned} O3 &\rightarrow \text{a point} \\ O5 &\rightarrow \text{2-cycle} \\ O7 &\rightarrow \text{wrapping a 4-cycle} \\ O9 &\rightarrow \text{6-cycle} \end{aligned} \quad (3.2.16)$$

In the later part, we perform the model building with $O3/O7$ -planes in the type IIB effective theory. One can add $D3/D7$ -branes to have consistent compactification, since the orientifold planes have negative charges which need to be canceled by D-brane and/or R-R charge.

Split cohomology

An orientifold projection on Calabi-Yau threefolds enables us to split the cohomology classes $H^{p,q}(CY_3)$ under the holomorphic involution σ^* . Namely, the involution decomposes both the cohomology group and their dimensionalities,

$$H^{p,q}(CY_3) = H_+^{p,q}(CY_3) \oplus H_-^{p,q}(CY_3), \quad h^{p,q} = h_+^{p,q} + h_-^{p,q}, \quad (3.2.17)$$

where $H_{+/-}^{p,q}(CY_3)$ contains even/odd forms under the action of σ^* . The basis of the resulting split cohomology classes are

$$\begin{aligned} \{\omega_\alpha\} &\in H_+^{1,1}(CY_3) \quad \alpha = 1, \dots, h_+^{1,1}, & \{\omega_a\} &\in H_-^{1,1}(CY_3) \quad a = 1, \dots, h_-^{1,1}, \\ \{\tilde{\omega}_\alpha\} &\in H_+^{2,2}(CY_3) \quad \alpha = 1, \dots, h_+^{1,1}, & \{\tilde{\omega}_a\} &\in H_-^{2,2}(CY_3) \quad a = 1, \dots, h_-^{1,1}, \\ \{\alpha_{\hat{\lambda}}, \beta^{\hat{\lambda}}\} &\in H_+^3(CY_3) \quad \hat{\lambda} = 1, \dots, h_+^{2,1}, & \{\alpha_\lambda, \beta^\lambda\} &\in H_-^3(CY_3) \quad \lambda = 0, \dots, h_-^{2,1}. \end{aligned} \quad (3.2.18)$$

In addition, the second equation in (3.2.15) yields to

$$\begin{aligned} h_-^{0,0} &= h_-^{3,3} = h_+^{3,0} = h_+^{0,3} = 0 \\ h_+^{0,0} &= h_+^{3,3} = h_-^{3,0} = h_-^{0,3} = 1. \end{aligned} \quad (3.2.19)$$

Orientifold action on fields

Here we discuss how the orientifold projections act on the fields shown in (3.1.1, 3.1.2). The worldsheet parity Ω interchanges the left- and right- movers, while the fermion operator $(-1)^{F_L}$ acts on the Ramond sectors which are constructed by fermions. As a combined effect, the orientifold projection requires the fields to transform under

$$\begin{aligned}\Omega(-1)^{F_L} g &\rightarrow g, & \Omega(-1)^{F_L} B_2 &\rightarrow -B_2, \\ \Omega(-1)^{F_L} \phi &\rightarrow \phi, & \Omega(-1)^{F_L} C_p &\rightarrow (-1)^{p/2} C_p.\end{aligned}\tag{3.2.20}$$

Note that in this way one can project out the odd state of the two gravitinos, thus half of the supersymmetry is broken by imposing the orientifold projection and we arrive in $N = 1$ supergravity.

3.2.3 Orientifold compactification with fluxes

Following the review of the basics of Calabi-Yau manifolds, we introduce the orientifold of type IIB superstring compactified on Calabi-Yau threefolds with non-vanishing geometric and non-geometric fluxes turned on. The additional fluxes give the possibility to spontaneously break the left $N = 1$ supersymmetry [128–132], and thus leads to non-supersymmetric models in phenomenology.

For vanishing fluxes, the massless spectrum comprises $h_+^{1,1}$ complexified Kähler moduli T_α , $h_-^{1,1}$ pure axionic moduli G^a , $h_-^{2,1}$ complex structure moduli U^i and $h_+^{2,1}$ abelian gauge fields A_j resulting from the dimensional reduction of the R-R four-form C_4 on three-cycles of the Calabi-Yau manifold [133]. In addition, the dilaton and the R-R 0-form give the chiral axio-dilaton, defined as $S = e^{-\phi} - iC_0$ in our conventions. Incorporating the constant fluxes H , F , Q and R , a twisted differential acting on p -forms is given by

$$\mathcal{D} = d - H \wedge - F \circ - Q \bullet - R \lrcorner, \tag{3.2.21}$$

where the operators entering in (3.2.21) act as

$$\begin{aligned}H \wedge &: p\text{-form} \rightarrow (p+3)\text{-form}, \\ F \circ &: p\text{-form} \rightarrow (p+1)\text{-form}, \\ Q \bullet &: p\text{-form} \rightarrow (p-1)\text{-form}, \\ R \lrcorner &: p\text{-form} \rightarrow (p-3)\text{-form}.\end{aligned}\tag{3.2.22}$$

For the different forms in a Calabi-Yau threefold this action can be specified by [130]

$$\begin{aligned}\mathcal{D}\alpha_\Lambda &= q_\Lambda{}^A \omega_A + f_{\Lambda A} \tilde{\omega}^A, & \mathcal{D}\beta^\Lambda &= \tilde{q}^{\Lambda A} \omega_A + \tilde{f}^\Lambda{}_A \tilde{\omega}^A, \\ \mathcal{D}\omega_A &= -\tilde{f}^\Lambda{}_A \alpha_\Lambda + f_{\Lambda A} \beta^\Lambda, & \mathcal{D}\tilde{\omega}^A &= \tilde{q}^{\Lambda A} \alpha_\Lambda - q_\Lambda{}^A \beta^\Lambda.\end{aligned}\tag{3.2.23}$$

with $\Lambda = 0, \dots, h^{2,1}$ and $A = 0, \dots, h^{1,1}$. For the H - and R -flux we further use the conventions

$$\begin{aligned} f_{\Lambda 0} &= r_{\Lambda}, & \tilde{f}^{\Lambda}_0 &= \tilde{r}^{\Lambda}, \\ q_{\Lambda}^0 &= h_{\Lambda}, & \tilde{q}^{\Lambda 0} &= \tilde{h}^{\Lambda}. \end{aligned} \quad (3.2.24)$$

We denote $\tilde{\omega}^0 = 1$, and $\omega_0 = \sqrt{g}d^6x/\mathcal{V}_{\mathcal{M}}$, where $\mathcal{V}_{\mathcal{M}} = \int_{\mathcal{M}} \sqrt{g}d^6x$ is the volume of the Calabi-Yau threefold \mathcal{M} .

Imposing the nilpotency condition of the form $\mathcal{D}^2 = 0$ we obtain the Bianchi identities for the fluxes,

$$\begin{aligned} 0 &= \tilde{q}^{\Lambda A} \tilde{f}^{\Sigma}_A - \tilde{f}^{\Lambda}_A \tilde{q}^{\Sigma A}, & 0 &= q_{\Lambda}^A f_{\Sigma A} - f_{\Lambda A} q_{\Sigma}^A, \\ 0 &= q_{\Lambda}^A \tilde{f}^{\Sigma}_A - f_{\Lambda A} \tilde{q}^{\Sigma A}, & 0 &= \tilde{f}^{\Lambda}_A q_{\Lambda}^B - f_{\Lambda A} \tilde{q}^{\Lambda B}, \\ 0 &= \tilde{f}^{\Lambda}_A f_{\Lambda B} - f_{\Lambda A} \tilde{f}^{\Lambda}_B, & 0 &= \tilde{q}^{\Lambda A} q_{\Lambda}^B - q_{\Lambda}^A \tilde{q}^{\Lambda B}. \end{aligned} \quad (3.2.25)$$

Imposing the orientifold projection, the invariant fluxes are

$$\begin{aligned} \mathfrak{F} &: & \mathfrak{f}_{\lambda}, & \tilde{\mathfrak{f}}^{\lambda}, \\ H &: & h_{\lambda}, & \tilde{h}^{\lambda}, \\ F &: & f_{\hat{\lambda}\alpha}, & \tilde{f}^{\hat{\lambda}}_{\alpha}, & f_{\lambda a}, & \tilde{f}^{\lambda}_a, \\ Q &: & q_{\hat{\lambda}}^a, & \tilde{q}^{\hat{\lambda}a}, & q_{\lambda}^{\alpha}, & \tilde{q}^{\lambda\alpha}, \\ R &: & r_{\hat{\lambda}}, & \tilde{r}^{\hat{\lambda}}. \end{aligned} \quad (3.2.26)$$

where $\lambda = 0, \dots, h^{2,1}_-$, $\hat{\lambda} = 1, \dots, h^{2,1}_+$, $\alpha = 1, \dots, h^{1,1}_+$ and $a = 1, \dots, h^{1,1}_-$. Note that in [134], the construction was restricted to the case $h^{2,1}_+ = 0$, whereas here we also consider $h^{2,1}_+ > 0$. In fact, as shown in [135], the fluxes with index λ contribute to an F-term scalar potential, while the fluxes with index $\hat{\lambda}$ contribute to a positive definite D-term potential. Later on, we will study the phenomenology raised from moduli stabilization with F-term and D-term.

For moduli stabilization, we allow all orientifold even fluxes, only subject to the Bianchi identities. The superpotential generating the F-term potential takes the form [129, 130]

$$W = \int_{\mathcal{M}} \left[\mathfrak{F} + \mathcal{D}\Phi_{\text{c}}^{\text{ev}} \right]_3 \wedge \Omega \quad (3.2.27)$$

with the complex multiform $\Phi_{\text{c}}^{\text{ev}} = iS - iG^a\omega_a - iT_{\alpha}\tilde{\omega}^{\alpha}$. Using (3.2.23) the superpotential can be further evaluated as

$$\begin{aligned} W &= -(\mathfrak{f}_{\lambda}X^{\lambda} - \tilde{\mathfrak{f}}^{\lambda}F_{\lambda}) + iS(h_{\lambda}X^{\lambda} - \tilde{h}^{\lambda}F_{\lambda}) \\ &\quad + iG^a(f_{\lambda a}X^{\lambda} - \tilde{f}^{\lambda}_aF_{\lambda}) - iT_{\alpha}(q_{\lambda}^{\alpha}X^{\lambda} - \tilde{q}^{\lambda\alpha}F_{\lambda}), \end{aligned} \quad (3.2.28)$$

where the periods X^λ, F_λ of the holomorphic 3-form Ω are computed from the tree-level cubic prepotential $F = \frac{1}{6}d_{ijk}X^iX^jX^k/X^0$ of the Calabi-Yau threefold². Specifically, Ω has the expansion $\Omega = X^\lambda\alpha_\lambda - F_\lambda\beta^\lambda$.

The tree-level Kähler potential in the large complex structure limit can be expressed as [133]

$$K = -\log\left(-i\int_{\mathcal{M}}\Omega\wedge\bar{\Omega}\right) - \log(S+\bar{S}) - 2\log\mathcal{V}. \quad (3.2.29)$$

Here $\mathcal{V} = \frac{1}{6}\kappa_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma$ denotes the volume of the Calabi-Yau threefold in Einstein frame. For future reference we also record the expansions of the Kähler and NS-NS 2-forms, respectively $J = e^{\phi/2}t^\alpha\omega_\alpha$ and $B_2 = b^a\omega_a$.

F-term potential

The F-term scalar potential V_F is computed in terms of the holomorphic superpotential W , the Kähler potential K as

$$V_F = \frac{M_{\text{Pl}}^4}{4\pi} e^K \left(G^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \quad (3.2.30)$$

in which I runs over all holomorphic fields of the theory while \bar{J} runs over all anti-holomorphic ones. Furthermore, the so-called Kähler covariant derivative $D_I W$ is

$$D_\alpha W = F_\alpha = \partial_\alpha W + K_\alpha W, \quad K_\alpha = \partial_\alpha K, \quad (3.2.31)$$

where $\partial_I K$ denotes the derivative of Kähler potential K with respect to the field labeled by I . In addition, the Kähler metric $G_{I\bar{J}}$ is computed from the Kähler potential K in the following way

$$G_{I\bar{J}} = \partial_I \partial_{\bar{J}} K. \quad (3.2.32)$$

D-term potential

Apart from the F-term potential, a D-term potential V_D is denoted in terms of $\text{Re}(f^{ab})$ (the real part of the inverse of the gauge kinetic function f_{ab}) and the auxiliary D-fields which satisfy $G_{\alpha\bar{\beta}} \bar{X}_\alpha^\beta = i\partial_\alpha D_\alpha$ with X_a^α the corresponding holomorphic Killing vectors [136, 137]. In the case of linear-transforming scalars and diagonal gauge kinetic function $f_{ab} = f_a \delta_{ab}$,

²The generically present subleading polynomial corrections to this cubic form will be considered later.

the D-term potential can be expressed as

$$V_D = \sum_a \frac{1}{8\text{Re}(f_a)} (K_\alpha T^\alpha \phi_\alpha + h.c.)^2, \quad (3.2.33)$$

with K_α the derivative of the Kähler potential K with respect to fields ϕ_α and T^a the representation matrices of the gauge symmetry.

$N = 1$ supergravity

The $N = 1$ supergravity theories in four dimensions can be characterized solely in terms of the Kähler potential K , a holomorphic superpotential W , gauge kinetic functions f_{ab} and possible Fayet-Iliopoulos terms ξ_a [136, 138]. Denoting by \star_4 the four-dimensional Hodge- \star -operator, the action takes the following general form³,

$$\mathcal{S} = \mathcal{S}_{kin} - \int_{R^{3,1}} (V_F + V_D) \star_4 1, \quad (3.2.34)$$

where \mathcal{S}_{kin} denotes the kinetic part while V_F and V_D stand for the F - and D -term potentials respectively.

In [135], it was explicitly shown that the F-term scalar potential resulting from the Kähler potential and superpotential reviewed above, is related to the one obtained via dimensional reduction of double field theory on a Calabi-Yau threefold with (non-)geometric fluxes⁴. Moreover, the potential is related to $N = 2$ gauged supergravity [139]. Taking the orientifold projection the scalar potential splits into three pieces

$$V = V_F + V_D + V_{\text{tad}}^{\text{NS}}, \quad (3.2.35)$$

where V_F is precisely the F-term scalar potential (3.2.30). $V_{\text{tad}}^{\text{NS}}$ is the NS-NS tadpole which will be canceled against the tension of the branes and orientifold planes, once R-R tadpole cancellation is taken into account. V_D is an additional D-term potential which takes the form of

$$V_D = -\frac{M_{\text{Pl}}^4}{2} \left[(\text{Im } \mathcal{N})^{-1} \right]^{\hat{\lambda}\hat{\sigma}} D_{\hat{\lambda}} D_{\hat{\sigma}} \quad (3.2.36)$$

resulting from the abelian gauge fields when $h_+^{2,1} > 0$. Adjusting the results in [135] to our conventions, the D-terms $D_{\hat{\lambda}}$ in Einstein frame are given by

$$D_{\hat{\lambda}} = \frac{1}{\mathcal{V}} \left[-r_{\hat{\lambda}} \left(e^\phi \mathcal{V} - \frac{1}{2} \kappa_{\alpha ab} t^\alpha b^a b^b \right) - q_{\hat{\lambda}}^a \kappa_{a\alpha b} t^\alpha b^b + f_{\hat{\lambda}\alpha} t^\alpha \right], \quad (3.2.37)$$

³Here for convenience we have set Planck scale $M_{\text{Pl}} = 1$.

⁴We will discuss in more detail about the source of non-geometric fluxes from DFT point of view in the next section.

whereas $\tilde{r}^{\hat{\lambda}} = \tilde{q}^{\hat{\lambda}a} = \tilde{f}^{\hat{\lambda}}_{\alpha} = 0$.

In [134], assuming $h_+^{2,1} = 0$, the F-term scalar potential V_F was investigated, in which W contained only $n + 1$ terms for a model with n superfields. This ansatz led to solutions where the fixed moduli, as well as the resulting moduli mass scales, could be expressed as simple quotients of fluxes. This allowed to gain parametric control over certain mass scales which was important for the realization of F-term axion monodromy inflation. A so-called scaling type minima, where the vacuum solutions are scaled by flux-values was introduced. All scaling vacua of this type were stable non-supersymmetric AdS minima, for which the existence of an uplift to Minkowski/de Sitter was not derived. In Chapter 6, we will show that for concrete examples Minkowski/de Sitter minima exist and feature nice scaling type behavior in string cosmology.

Non-geometric S-dual P -form fluxes

Adding the non-geometric Q -fluxes, the superpotential (3.2.28) is no longer covariant under S-duality transformations. It has been shown in [140] that this covariance can be restored by including non-geometric P -fluxes, which transform together with the Q -fluxes as a doublet of the $SL(2, \mathbb{Z})$ duality group. The additional P -fluxes are considered as the S-duals of Q -fluxes. Similar to the Q -flux, it is defined as a map

$$P \bullet : p\text{-form} \rightarrow (p - 1)\text{-form}, \quad (3.2.38)$$

and the action of P on the symplectic basis is specified by

$$\begin{aligned} -P \bullet \alpha_{\Lambda} &= p_{\Lambda}^A, & -P \bullet \beta^{\Lambda} &= \tilde{p}^{\Lambda A} \omega_A, \\ -P \bullet \omega_A &= 0, & -P \bullet \tilde{\omega}^A &= -p^{\Lambda A} \alpha_{\Lambda} + p_{\Lambda}^A \beta^{\Lambda}. \end{aligned} \quad (3.2.39)$$

Incorporating the S-duality and geometric moduli G^a , an extended superpotential is derived in [134],

$$W' = \int_{\mathcal{M}} [\mathfrak{F} + \mathcal{D}\Phi_c^{\text{ev}} + T_{\alpha} S(P \bullet \tilde{\omega}^{\alpha}) + \frac{1}{2} \kappa_{abc} G^b G^c (P \bullet \tilde{\omega}^{\alpha})]_3 \wedge \Omega_3, \quad (3.2.40)$$

which after integrations yields to⁵

$$W' = W + \left(S T_{\alpha} + \frac{1}{2} \kappa_{abc} G^b G^c \right) (p_{\lambda}^{\alpha} X^{\lambda} - \tilde{p}^{\lambda \alpha} F_{\lambda}), \quad (3.2.41)$$

where W is shown in (3.2.28). The Bianchi identities in this case were also discussed in [140]. For our examples, we notice that in general the only non-trivial constraint with

⁵In Chapter 6, we will restrict our attention to the examples with $h_-^{1,1} = 0$ so that the geometric moduli (G^a) contribution to the scalar potential is absent.

Q -fluxes comes from the last equation of (3.2.25)

$$\tilde{q}^{\Lambda A} h_{\Lambda} - q_{\Lambda}^A \tilde{h}^{\Lambda} = 0. \quad (3.2.42)$$

Performing one S-duality transformation, this leads to the Bianchi identity regards to P -flux in the form of

$$\tilde{p}^{\Lambda A} \mathfrak{f}_{\Lambda} - p_{\Lambda}^A \tilde{\mathfrak{f}}^{\Lambda} = 0. \quad (3.2.43)$$

Here we have used that both (P, Q) and (\mathfrak{F}, H) fluxes transform as an $SL(2, \mathbb{Z})$ doublet.

Part II

String Applications

4

Non-geometry in Heterotic DFT and EFT

Recall that successive T-dualities applied to a closed string background with constant H -flux led to a flux background chain (1.0.2). DFT incorporates the non-geometric backgrounds as the T-dual of geometric ones in one framework. In [61, 141], the authors determined the T-dual of a bosonic string compactified on a three-torus T^3 with constant H flux turned on. It has been shown in [141] that in DFT frameworks applying two or three T-dualities to H -flux background led to non-geometric Q - or R -flux background respectively. The non-geometry shows up in the appearance of winding coordinates in the transition functions for the Q -flux and in the background itself for the R -flux. Therefore, a Q -flux background is locally geometric but not globally, whereas a R -flux background is non-geometric even locally.

However, to our knowledge, it is not clear what the heterotic T-dual of a constant gauge flux background is. In this chapter we study the T-dual of a heterotic string compactified on a two-torus T^2 with a constant gauge flux turned on. In the framework of heterotic DFT, we will show that indeed after one T-duality, one gets a non-geometric gauge flux background that is in many ways analogous to the Q -flux background. It is locally still geometric and the non-geometry appears in the transition functions in the sense that there appears a dependence on a winding coordinate. Moreover, one can perform a field redefinition to a non-geometric frame in which the fundamental fields are a dual metric \tilde{g}_{ij} , a bi-vector β^{ij} and a gauge one-vector \tilde{A}^i . We will see that one gets a chain of gauge fluxes $G_{ij} \rightarrow J^i_j \rightarrow \tilde{G}^{ij}$, where the latter two are non-geometric. We trace back that, in this case, the non-geometry arises due to the α' corrections to the T-duality rules [142, 143].

Furthermore, we recall that in generalized geometry framework, an $O(D, D)$ induced field redefinition can be described by the differential geometry of a corresponding Lie algebroid [91, 92]. This connected the dual supergravity action in the non-geometric frame with the standard supergravity action in the geometric frame. It also has been shown that the non-geometric actions, derived from rigid $O(D, D)$ transformations, are related to the DFT action under the strong constraint. Here we are interested in whether a similar analysis can be generalized to the heterotic case with global $O(D, D + n)$ transformations. We will show that an $O(D, D + n)$ induced field redefinition can still be described by the differential geometry of a corresponding Lie algebroid. We will explicitly present the

corrections due to the existence of the gauge field. As for the original version, the local symmetries of the redefined action are only the redefined versions of diffeomorphisms, B -field and A -field gauge transformation. This implies that a single such action cannot globally describe non-geometric backgrounds, which need winding coordinates to appear either in the transition functions (Q -flux) or in the background itself (R -flux).

4.1 Non-geometric backgrounds of heterotic DFT

In this section, we will perform in detail successive T-dualities on a toroidal constant gauge flux background and show how a field redefinition to a non-geometric frame simplifies the description of the T-dual backgrounds. We will derive explicitly the form of the relevant heterotic fluxes and comment on the consequences for a potential non-associativity and for S-duality to the type I string.

4.1.1 T-duality of a constant gauge flux background

Recall that under a global $h \in O(D, D + n)$ transformation the coordinates and the generalized metric transform as

$$H' = h^t H h, \quad X' = h X, \quad \partial' = (h^t)^{-1} \partial. \quad (4.1.1)$$

Here we consider a 2-torus background with a flat metric $g_{ij} = \delta_{ij}$, vanishing Kalb-Ramond field B and a constant abelian gauge flux G_{ij} . For the corresponding single gauge potential $A^{(1)} = A$ we choose

$$A_1 = f y, \quad A_2 = 0. \quad (4.1.2)$$

This gives the field strength

$$G_{12} = -(\partial_1 A_2 - \partial_2 A_1) = f. \quad (4.1.3)$$

On the T^2 background the coordinates are periodically identified by $(x, y) \sim (x + 2\pi, y) \sim (x, y + 2\pi)$. For the gauge field to be well defined globally, one needs a non-trivial transition function between the two patches $P = [0, 2\pi)$ and $Q = (0, 2\pi]$. In the patch P we have $A_1^{(P)} = f y$ while in the patch Q the gauge field is $A_1^{(Q)} = f(y - 2\pi)$. These two patches can be glued smoothly together by a gauge transformation $A_1^{(Q)} = A_1^{(P)} + \partial_1 \lambda^{(PQ)}$ with

$$\lambda^{(PQ)} = -2\pi f x. \quad (4.1.4)$$

The generalized metric for this background in patch P takes the form

$$\mathcal{H}_{MN}^{(P)} = \left(\begin{array}{cc|cc|c} 1 & 0 & -\frac{(fy)^2}{2} & 0 & -(fy) \\ 0 & 1 & 0 & 0 & 0 \\ \hline -\frac{(fy)^2}{2} & 0 & 1 + (fy)^2 + \frac{(fy)^4}{4} & 0 & (fy) + \frac{(fy)^3}{2} \\ 0 & 0 & 0 & 1 & 0 \\ \hline -(fy) & 0 & (fy) + \frac{(fy)^3}{2} & 0 & 1 + (fy)^2 \end{array} \right). \quad (4.1.5)$$

The transition to patch Q is given by conjugation with an appropriate $O(D, D+n)$ matrix $\mathcal{T}_{(PQ)}$, i.e.

$$\mathcal{H}^{(Q)} = \mathcal{T}_{(PQ)}^T \mathcal{H}^{(P)} \mathcal{T}_{(PQ)} \quad (4.1.6)$$

which in our case takes the form

$$\mathcal{T}_{(PQ)} = \left(\begin{array}{cc|cc|c} 1 & 0 & -\frac{(2\pi f)^2}{2} & 0 & 2\pi f \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & -2\pi f & 0 & 1 \end{array} \right). \quad (4.1.7)$$

In analogy to generalized geometry, such a matrix can be called an “ A -transform”. Note that this is consistent with the discussion in [90], where the transition matrix was calculated via the vielbeins in the two patches as $\mathcal{T}_{(PQ)} = E_{(P)}^{-1} E_{(Q)}$.

Applying a T-duality in the x -direction, which in heterotic DFT can be implemented by conjugation $\mathcal{H}' = \mathcal{T}_1^T \mathcal{H} \mathcal{T}_1$ with the special $O(2, 3)$ transformation¹

$$\mathcal{T}_1 = \left(\begin{array}{cc|cc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array} \right), \quad (4.1.8)$$

¹The upper 4×4 dimensional part of the metric is identical to the T-duality transformation for bosonic DFT.

we obtain in patch P

$$\mathcal{H}'^{(P)} = \left(\begin{array}{cc|cc|c} 1 + (fy)^2 + \frac{(fy)^4}{4} & 0 & -\frac{(fy)^2}{2} & 0 & (fy) + \frac{(fy)^3}{2} \\ 0 & 1 & 0 & 0 & 0 \\ \hline -\frac{(fy)^2}{2} & 0 & 1 & 0 & -(fy) \\ 0 & 0 & 0 & 1 & 0 \\ \hline (fy) + \frac{(fy)^3}{2} & 0 & -(fy) & 0 & 1 + (fy)^2 \end{array} \right). \quad (4.1.9)$$

One can directly read off the new metric, Kalb-Ramond field and the gauge field as

$$g'^{(P)} = \begin{pmatrix} \frac{1}{1+(fy)^2 + \frac{(fy)^4}{4}} & 0 \\ 0 & 1 \end{pmatrix}, \quad B'^{(P)} = 0, \quad A'^{(P)} = \begin{pmatrix} -\frac{(fy)}{1 + \frac{(fy)^2}{2}} \\ 0 \end{pmatrix}. \quad (4.1.10)$$

Note that after one T-duality we get a metric and a gauge field, where, as in the Q -flux background, there appears a non-trivial functional dependence in the denominators. Moreover, we check step by step that our results can be confirmed from α' corrected heterotic Buscher rules for T-duality along a single direction together with B -fields given in [143]. We give details of T-duality rules following from the heterotic DFT construction in appendix B.

Accordingly, we construct the new transition matrix to patch Q as

$$\mathcal{T}'_{(PQ)} = \mathcal{T}_1^T \mathcal{T}_{(PQ)} \mathcal{T}_1 = \left(\begin{array}{cc|cc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline -\frac{(2\pi f)^2}{2} & 0 & 1 & 0 & 2\pi f \\ 0 & 0 & 0 & 1 & 0 \\ \hline -2\pi f & 0 & 0 & 0 & 1 \end{array} \right) \quad (4.1.11)$$

which is no longer a usual A -transform, i.e. a gauge transformation. This observation and the appearance of strange denominators already indicate that we are dealing here rather with a non-geometric background (like the Q -flux for bosonic DFT).

In analogy to bosonic DFT, one introduce a field redefinition so that the generalized metric can be parameterized by a new metric \tilde{g}_{ij} , a bi-vector \tilde{C}^{ij} and a (one-)vector \tilde{A}^i as

$$\mathcal{H}_{MN} = \begin{pmatrix} \tilde{g}^{ij} + \tilde{C}^{ki} \tilde{g}_{kl} \tilde{C}^{lj} + \tilde{A}^i{}_\gamma \tilde{A}^{j\gamma} & -\tilde{g}_{jk} \tilde{C}^{ki} & \tilde{C}^{ki} \tilde{g}_{kl} \tilde{A}^l{}_\beta + \tilde{A}^i{}_\beta \\ -\tilde{g}_{ik} \tilde{C}^{kj} & \tilde{g}_{ij} & -\tilde{g}_{ik} \tilde{A}^k{}_\beta \\ \tilde{C}^{kj} \tilde{g}_{kl} \tilde{A}^l{}_\alpha + \tilde{A}^j{}_\alpha & -\tilde{g}_{jk} \tilde{A}^k{}_\alpha & \delta_{\alpha\beta} + \tilde{A}^k{}_\alpha \tilde{g}_{kl} \tilde{A}^l{}_\beta \end{pmatrix}, \quad (4.1.12)$$

where $\tilde{C}^{ij} = \beta^{ij} + \frac{1}{2} \tilde{A}^i{}_\alpha \tilde{A}^{j\alpha}$, and β^{ij} as the antisymmetric bi-vector appearing also in

bosonic DFT. The heterotic generalized vielbein reads

$$E^A_M = \begin{pmatrix} \tilde{e}_a^i & 0 & 0 \\ -\tilde{e}_k^a \tilde{C}^{ki} & \tilde{e}_i^a & -\tilde{e}_k^a \tilde{A}^k_\beta \\ \tilde{A}^{i\alpha} & 0 & \delta^\alpha_\beta \end{pmatrix}. \quad (4.1.13)$$

Comparing (4.1.9) with the form of the generalized metric in the so-called non-geometric frame (4.1.12), one can read off

$$\tilde{g}^{(P)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{A}^{(P)} = \begin{pmatrix} fy \\ 0 \end{pmatrix}, \quad (4.1.14)$$

with $\beta^{ij} = 0$. It is shown that in this frame the T-dual configuration takes a very simple form. Moreover, using (4.1.11) one can find the metric and the one-vector in patch Q

$$\tilde{g}^{(Q)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{A}^{(Q)} = \begin{pmatrix} f(y - 2\pi) \\ 0 \end{pmatrix}. \quad (4.1.15)$$

Recall that the T-duality suggests $x \rightarrow \tilde{x}$ in the gauge transformation (4.1.4), the “gauge” transformation connecting the one-vectors in patch P and Q becomes

$$\tilde{A}^{(Q)} = \tilde{A}^{(P)} + \tilde{\partial}^1 \tilde{\lambda}^{(PQ)} \quad \text{with} \quad \tilde{\lambda}^{(PQ)} = -2\pi f \tilde{x}. \quad (4.1.16)$$

Note that the transition function in this non-geometric frame contains a winding coordinate, so that indeed this T-dual background is globally non-geometric, as it is in the Q -flux background for bosonic DFT. The only difference is that the latter requires a T-duality in two-directions in order to generate it from a constant H -flux background. Finally, we give the new flux in this T-dual background

$$J^1_2 = -\partial_2 \tilde{A}^1 = -f. \quad (4.1.17)$$

Applying another T-duality in the y direction changes $y \rightarrow \tilde{y}$ in the generalized metric (4.1.9), so that in the non-geometric frame one obtains

$$\tilde{g}^{(P)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{A}^{(P)} = \begin{pmatrix} f\tilde{y} \\ 0 \end{pmatrix}, \quad (4.1.18)$$

and similarly in patch Q . Therefore, like in the R -flux background, this configuration is already locally non-geometric, characterized by a non-geometric flux

$$\tilde{G}^{12} = -(\tilde{\partial}^1 \tilde{A}^2 - \tilde{\partial}^2 \tilde{A}^1) = f. \quad (4.1.19)$$

Of course, at this stage the forms of the new non-geometric fluxes J^i_j and \tilde{G}^{ij} remain as a suggestion. In the following subsection, we will concretely derive the complete form of

this new kind of fluxes from the vielbein (4.1.13).

4.1.2 The fluxes of heterotic DFT

First, we recall the definition of heterotic fluxes in DFT formulation

$$\mathcal{F}_{ABC} = E_{CM} \mathcal{L}_{E_A} E_B^M = \Omega_{ABC} + \Omega_{CAB} - \Omega_{BAC}. \quad (4.1.20)$$

In order to treat geometric and non-geometric components at the same time as in [66], we use the extended form of the generalized vielbein

$$E^A_M = \begin{pmatrix} e_a^i & -e_a^k C_{ki} & -e_a^k A_{k\beta} \\ -e^a_k \tilde{C}^{ki} & e^a_i + e^a_k \tilde{C}^{kj} C_{ji} & -e^a_k \tilde{A}^k_\beta \\ \tilde{A}^{i\alpha} & A_i^\alpha & \delta^\alpha_\beta \end{pmatrix} \quad (4.1.21)$$

which combines (2.3.44) and (4.1.13) into one object. Recall that $\eta_{AB} = E_A^M E_{MB}$ implies that the Weitzenböck connection satisfies $\Omega_{ABC} = -\Omega_{ACB}$. However, one can show that this relation ceases to be satisfied with the full vielbein (4.1.21). Therefore, in the following we present the geometric fluxes for the physically relevant case of $\tilde{A}^i_\alpha = 0$ and the non-geometric fluxes for $A_i^\alpha = 0$. In addition, for simplicity here we will work in a holonomic basis, the rather lengthy generalizations to a non-holonomic basis can be found in appendix C.

Note that the components of the derivatives $D_A = E_A^M D_M$ are

$$\begin{aligned} \tilde{D}^i &= \tilde{\partial}^i + \tilde{C}^{im} C_{mn} \tilde{\partial}^n - \tilde{C}^{im} \partial_m - \tilde{A}^{i\gamma} \partial_\gamma, \\ D_i &= \partial_i - C_{im} \tilde{\partial}^m - A_i^\gamma \partial_\gamma, \\ D_\alpha &= \partial_\alpha + A_{m\alpha} \tilde{\partial}^m + \tilde{A}^m_\alpha \partial_m. \end{aligned} \quad (4.1.22)$$

For all three indices being of normal or winding type we get the fluxes H, F, Q and R including corrections depending on the gauge fields A and \tilde{A} . In terms of the derivatives (4.1.22), for $\tilde{A}^i_\alpha = 0$, the geometric fluxes can be expressed as

$$\begin{aligned} \mathcal{H}_{ijk} &= -3D_{[i} B_{jk]} + 3D_{[i} A_{j\gamma} A_{k]}^\gamma \\ F^k_{ij} &= -\tilde{D}^k B_{ij} + \tilde{D}^k A_{[i\gamma} A_{j]}^\gamma - 2D_{[i} \beta^{km} A_{j]\gamma} A_m^\gamma - 2\beta^{km} D_{[i} C_{m]j}. \end{aligned} \quad (4.1.23)$$

For $A_i^\alpha = 0$, the non-geometric fluxes take the form

$$\begin{aligned} Q_k^{ij} &= -D_k \beta^{ij} + D_k \tilde{A}^{[i\gamma} \tilde{A}^{j]}_\gamma - \tilde{C}^{[im} \tilde{C}^{j]n} D_k B_{mn} - 2D^{[i} B_{km} \tilde{C}^{j]m} \\ R^{ijk} &= -3\tilde{D}^{[i} \beta^{jk]} + 3\tilde{D}^{[i} \tilde{A}^{j\gamma} \tilde{A}^{k]}_\gamma + 3\tilde{C}^{[im} \tilde{D}^{j} B_{mn} \tilde{C}^{k]n} \end{aligned} \quad (4.1.24)$$

As expected, for $A_i^\alpha = \tilde{A}^i_\alpha = 0$, these expressions are consistent with the ones derived

in [66, 67, 122]. Due to the extra gauge coordinates y^α in heterotic DFT, there exist new types of fluxes. Choosing at least one index of \mathcal{F}_{ABC} to be a gauge index, the antisymmetry $\Omega_{ABC} = -\Omega_{ACB}$ in all indices lead us to set either $\beta^{ij} = \tilde{A}^i_\alpha = 0$ or $B_{ij} = A_i^\alpha = 0$. Of course, one can choose these constraints independently for each direction (ij) or (i) , respectively.

In the following, we present the result for choosing the same set of conditions for all directions. Therefore, in the geometric frame $\beta^{ij} = \tilde{A}^i_\alpha = 0$, we obtain the following three types of non-vanishing gauge fluxes

$$\begin{aligned} G_{\alpha ij} &= -2D_{[i}A_{j]\alpha} - D_\alpha B_{ij} + D_\alpha A_{[i}{}^\gamma A_{j]\gamma}, \\ J^j_{\alpha i} &= \tilde{\partial}^j A_{i\alpha}, \\ K_{\alpha\beta i} &= 2D_{[\alpha}A_{i\beta]}. \end{aligned} \tag{4.1.25}$$

Implementing the strong constraint via $\tilde{\partial}^i = \partial_\alpha = 0$, the first flux reduces to the familiar form of the field strength (2.3.28) for an abelian field. In the non-geometric frame $B_{ij} = A_i^\alpha = 0$, the non-vanishing fluxes take the form

$$\begin{aligned} J^j_{\alpha i} &= -\partial_i \tilde{A}^j_\alpha, \\ \tilde{G}_\alpha{}^{ij} &= -2\tilde{D}^{[i}\tilde{A}^{j]}\alpha - D_\alpha \beta^{ij} + D_\alpha \tilde{A}^{[i\gamma} \tilde{A}^{j]}\gamma, \\ \tilde{K}^{\alpha\beta i} &= 2D^{[\alpha} \tilde{A}^{i\beta]}. \end{aligned} \tag{4.1.26}$$

Hence, the flux $J^j_{\alpha i}$ in the non-geometric frame is indeed the flux we encountered in the previous section after applying one T-duality. Similarly, reducing $\tilde{G}_\alpha{}^{ij}$ for $\partial_i = \partial_\alpha = 0$, one obtains

$$\tilde{G}_\alpha{}^{ij} = -2\tilde{\partial}^{[i}\tilde{A}^{j]}\alpha, \tag{4.1.27}$$

with the gauge flux of \tilde{A} found in the background after applying two T-dualities.

For a non-holonomic basis, one finds the commutators

$$\begin{aligned} [\partial_a, \partial_b] &= f^c{}_{ab} \partial_c, \quad \text{with} \quad f^c{}_{ab} := e_i{}^c (\partial_a e_b{}^i - \partial_b e_a{}^i), \\ [\tilde{\partial}^a, \tilde{\partial}^b] &= \tilde{f}^c{}^{ab} \tilde{\partial}^c, \quad \text{with} \quad \tilde{f}^c{}^{ab} := e_c{}^i (\tilde{\partial}^a e_i{}^b - \tilde{\partial}^b e_i{}^a). \end{aligned} \tag{4.1.28}$$

providing correction terms to the fluxes shown above. The resulting rather lengthy expressions for these fluxes can be found in appendix C.

The upshot of the explicit analysis of this section is that, for the heterotic string, the T-dual of the constant gauge flux background on a flat geometry is a *non-geometric* background. Therefore, we confirm that the concept of non-geometry does not only apply to closed string three-form backgrounds but also to gauge flux backgrounds. Moreover, we show that for the description of these T-dual backgrounds, it is appropriate to change to a non-geometric frame, where in particular the gauge 1-form $A = A_i dx^i$ is replaced by a

gauge 1-vector $\tilde{A} = \tilde{A}^i \partial_i$.

4.1.3 Comment on S-duality

Let us now consider the $SO(32)$ heterotic string compactified on a two-torus with constant abelian gauge flux $F = F_{12}$. This configuration is known to be S-dual to the Type I string [144] compactified on a two-torus where the $D9$ -brane carries the same gauge flux F . Applying a T-duality in the y -direction to this latter configuration yields the Type I' string with a $D8$ -brane at an angle with respect to the $O8$ -planes. One might ask whether there exist an S-dual to this configuration. The answer to this question is not obvious, as in the heterotic string theory there are no 8-branes. However, recall that we have just seen that the T-dual to the $SO(32)$ heterotic string with gauge flux is a non-geometric background of the $E_8 \times E_8$ heterotic string carrying flux $J = J^1_2$. Therefore, by completing the diagram as shown in Figure 4.1 we are led to the conjecture that the S-dual of the $D8$ -brane at angle in Type I' is a non-geometric J -flux background of the heterotic string.

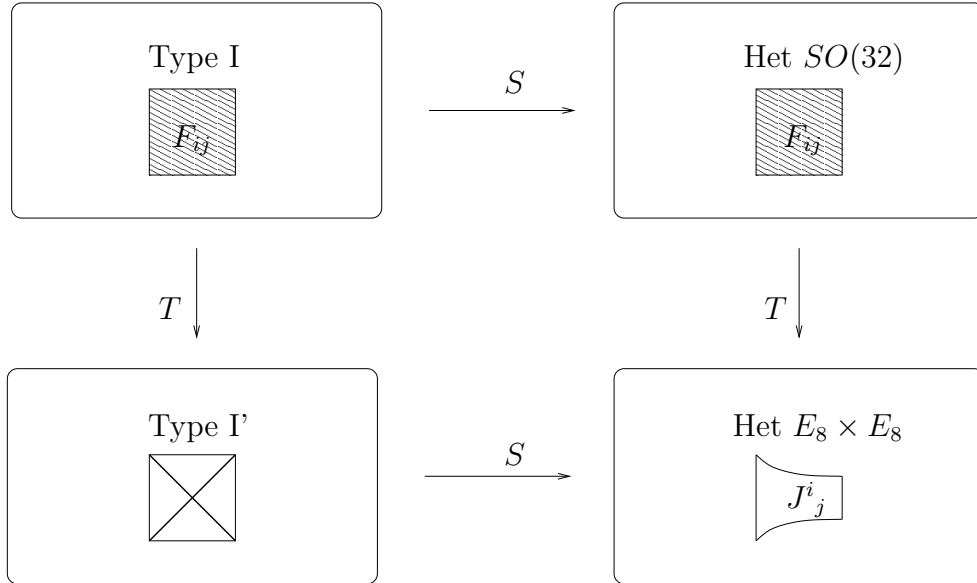


Figure 4.1: S - and T -duality between Type I and heterotic string.

4.2 A Lie algebroid for heterotic field redefinitions

Recall that in [92] a general structure of $O(D, D)$ induced field definitions was clarified in the framework of generalized geometry. The field redefinition simplified the description of non-geometric backgrounds in DFT as well. The two main results were that for every such

field redefinition, one can associate a corresponding Lie algebroid so that the redefined supergravity action in the non-geometric frame is governed by the differential geometry of that Lie algebroid.

In this section, we show that this picture also holds for the heterotic case, i.e. to every $O(D, D+n)$ induced field redefinition one can associate a corresponding Lie algebroid, so that in the new field variables, the heterotic action is governed by the differential geometry of that Lie algebroid. For the definition of Lie algebroid, please consult appendix 4.2.1. In addition, we will show that the non-geometric frame (4.1.12) also fits into this scheme. Since the story is very similar, we will be rather brief here and refer to [92] for more information on Lie algebroids and their differential geometry.

4.2.1 Lie algebroids

A Lie algebroid is specified by three pieces of information:

- a vector bundle E over a manifold M ,
- a bracket $[\cdot, \cdot]_E : E \times E \rightarrow E$, and
- a homomorphism $\rho : E \rightarrow TM$ called the anchor.

Similar to the usual Lie bracket, one requires the bracket $[\cdot, \cdot]_E$ to satisfy a Leibniz rule. Denoting functions by $f \in \mathcal{C}^\infty(M)$ and sections of E by s_i , this reads

$$[s_1, fs_2]_E = f[s_1, s_2]_E + \rho(s_1)(f)s_2, \quad (4.2.1)$$

where $\rho(s_1)$ is a vector field which acts on f as a derivation. If in addition the bracket $[\cdot, \cdot]_E$ satisfies a Jacobi identity

$$[s_1, [s_2, s_3]_E]_E = [[s_1, s_2]_E, s_3]_E + [s_2, [s_1, s_3]_E]_E, \quad (4.2.2)$$

then $(E, [\cdot, \cdot]_E, \rho)$ is called a Lie algebroid.

Moreover, any Lie algebroid can be equipped with a nilpotent exterior derivative as follows

$$\begin{aligned} d_E \theta^*(s_0, \dots, s_n) &= \sum_{i=0}^n (-1)^i \rho(s_i) \theta^*(s_0, \dots, \hat{s}_i, \dots, s_n) \\ &\quad + \sum_{i < j} (-1)^{i+j} \theta^*([s_i, s_j]_E, s_0, \dots, \hat{s}_i, \dots, \hat{s}_j, \dots, s_n), \end{aligned} \quad (4.2.3)$$

where $\theta^* \in \Gamma(\Lambda^n E^*)$ is the analog of an n -form on the Lie algebroid and \hat{s}_i denotes the omission of that entry. The Jacobi identity of the bracket $[\cdot, \cdot]_E$ implies that (4.2.3) satisfies

$(d_E)^2 = 0$. The anchor property and the corresponding formula for the de Rahm differential allow to compute

$$\begin{aligned} \left((\Lambda^{n+1} \rho^*) (d_E \theta^*) \right) (X_0, \dots, X_n) &= (d_E \theta^*) (\rho^{-1}(X_0), \dots, \rho^{-1}(X_n)) \\ &= d((\Lambda^n \rho^*)(\theta^*)) (X_0, \dots, X_n) \end{aligned} \quad (4.2.4)$$

with the dual anchor $\rho^* = (\rho^t)^{-1}$ and for sections $X_i \in \Gamma(TM)$. The relation (4.2.4) describes how exact terms translate in general.

4.2.2 $O(D, D + n)$ -induced field redefinition

In abelian heterotic generalized geometry, one considers a D-dimensional manifold M with usual coordinates x^i and equipped with a generalized bundle $E = TM \oplus T^*M \oplus V$, whose sections are formal sums $\xi + \tilde{\xi} + \lambda$ of vectors, $\xi = \xi^i(x) \partial_i$, one-forms, $\tilde{\xi} = \tilde{\xi}_i(x) dx^i$ and gauge transformations, $\lambda = (\lambda_1(x), \dots, \lambda_n(x))$, of $U(1)^n$. On this bundle, one defines a generalized metric \mathcal{H}_{MN} taking the familiar form (2.3.38) in terms of the fundamental fields g_{ij} , B_{ij} and A_i^α . An $O(D, D + n)$ transformation \mathcal{M} acts on the generalized metric via conjugation, i.e.

$$\hat{H}(\hat{g}, \hat{B}, \hat{A}) = \mathcal{M}^t H(g, B, A) \mathcal{M} \quad (4.2.5)$$

and thus defines a field redefinition

$$(g, B, A) \longrightarrow (\hat{g}, \hat{B}, \hat{A}). \quad (4.2.6)$$

The heterotic action in terms of the fields (g, B, A) is the heterotic supergravity action (2.3.27). Regarding to the action in the new field variables $(\hat{g}, \hat{B}, \hat{A})$, we study the organizing principle in the heterotic framework. To proceed, we provide a general $O(D, D + n)$ transformation matrix \mathcal{M}

$$\mathcal{M} = \begin{pmatrix} a & b & m \\ c & d & n \\ p & q & z \end{pmatrix}, \quad (4.2.7)$$

which leaves the η metric (2.3.32) invariant,

$$\mathcal{M}^t \eta \mathcal{M} = \eta. \quad (4.2.8)$$

This leads to six independent constraints on the submatrices

$$\begin{aligned}
c^t a + a^t c + p^t p &= 0 \\
c^t b + a^t d + p^t q &= 1 \\
c^t m + a^t n + p^t z &= 0 \\
d^t b + b^t d + q^t q &= 0 \\
d^t m + b^t n + q^t z &= 0 \\
n^t m + m^t n + z^t z &= 1.
\end{aligned} \tag{4.2.9}$$

Applying (4.2.5), one can read off the induced field redefinition as follows. The upper-left component of $\hat{H}(\hat{g}, \hat{B}, \hat{A})$ reads

$$\hat{H}(\hat{g}, \hat{B}, \hat{A})_{ul} = \left[a - A p - (g + B + \tfrac{1}{2} A^2) c \right]^t g^{-1} \left[a - A p - (g + B + \tfrac{1}{2} A^2) c \right]. \tag{4.2.10}$$

Comparing with the general form of the generalized metric, this gives \hat{g}^{-1} . We construct

$$\hat{g} = (\gamma^{-1}) g (\gamma^{-1})^t \tag{4.2.11}$$

where the matrix

$$\gamma = a - A p - (g + B + \tfrac{1}{2} A^2) c. \tag{4.2.12}$$

In order to consider the redefined Kalb-Ramond field \hat{B} (which is contained in \hat{C}), we read the upper-middle component of the redefined generalized metric

$$\hat{H}(\hat{g}, \hat{B}, \hat{A})_{um} = 1 + \left[a - A p - (g + B + \tfrac{1}{2} A^2) c \right]^t g^{-1} \left[b - A q - (g + B + \tfrac{1}{2} A^2) d \right]. \tag{4.2.13}$$

Compare it with $\hat{H}_{um} = -\hat{g}^{-1} \hat{C}$, we find

$$\hat{C} = (\gamma^{-1}) \mathfrak{C} (\gamma^{-1})^t, \quad \text{with} \quad \mathfrak{C} = \delta \gamma^t - g, \tag{4.2.14}$$

where the matrix

$$\delta = -b + A q + (g + B + \tfrac{1}{2} A^2) d. \tag{4.2.15}$$

To determine the $O(D, D+n)$ induced field redefinition for the gauge field A , we look into the upper-right element of the generalized metric

$$\hat{H}(\hat{g}, \hat{B}, \hat{A})_{ur} = \left[a - A p - (g + B + \tfrac{1}{2} A^2) c \right]^t g^{-1} \left[m - A z - (g + B + \tfrac{1}{2} A^2) n \right], \tag{4.2.16}$$

and identify it with $-\hat{g}^{-1}\hat{A}$. We obtain

$$\hat{A} = (\gamma^{-1}) \mathfrak{A} \quad (4.2.17)$$

with

$$\mathfrak{A} = -m + A z + \left(g + B + \frac{1}{2} A\right) n. \quad (4.2.18)$$

According to \mathfrak{C} and \mathfrak{A} , one can define also a new B -field \mathfrak{B} ,

$$\hat{B} = (\gamma^{-1}) \mathfrak{B} (\gamma^{-1})^t \quad \text{with} \quad \mathfrak{B} = \mathfrak{C} - \frac{1}{2} \mathfrak{A} \otimes \mathfrak{A}. \quad (4.2.19)$$

The field redefinition is of a very peculiar form, where the matrix γ plays a prominent role. The structure of the field redefinitions of g and B is quite analogous to [92], whereas containing new gauge field dependent corrections in γ and δ . It is straightforward to proceed as in [92] and to identify

$$\rho = (\gamma^{-1})^t \quad (4.2.20)$$

as the anchor map of a Lie algebroid (see appendix 4.2.1). This Lie algebroid lives on the tangent bundle itself, i.e. $E = TM$ and the anchor map $\rho : E \rightarrow TM$ acts on a vector field $X = X^i \partial_i \in E$ as²

$$\rho(X) = (\rho^i_j X^j) \partial_i = X^i (\rho^t)_i^j \partial_j = X^i D_i, \quad (4.2.21)$$

where we defined the partial derivative for the Lie algebroid as

$$D_i = (\rho^t)_i^j \partial_j. \quad (4.2.22)$$

The bracket $\llbracket \cdot, \cdot \rrbracket$ on $E = TM$ is defined as

$$\llbracket X, Y \rrbracket = \left(X^j D_j Y^k - Y^j D_j X^k + X^i Y^j F_{ij}{}^k \right) \partial_k. \quad (4.2.23)$$

with the structure constants

$$F_{ij}{}^k = (\rho^{-1})^k_m \left(D_i (\rho^t)_j^m - D_j (\rho^t)_i^m \right). \quad (4.2.24)$$

Indeed, this bracket satisfies the homomorphism property

$$\rho(\llbracket X, Y \rrbracket) = [\rho(X), \rho(Y)]. \quad (4.2.25)$$

²Here we present the relations in a holonomic basis. For the non-holonomic case, we refer to [92].

Furthermore, by construction, the new bracket $\llbracket \cdot, \cdot \rrbracket$ satisfies the Jacobi identity (4.2.2) as well as the Leibniz rule (4.2.1). Thus, for every $O(D, D+n)$ induced field redefinition we have associated a corresponding Lie algebroid. The true power of this formal approach will become clear in the next section.

4.2.3 The redefined heterotic action

Recall that the NS-sector of the heterotic DFT action is

$$\mathcal{S} = \int dx \sqrt{g} e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} - \frac{1}{4} G^{ij\alpha} G_{ij\alpha} \right), \quad (4.2.26)$$

with the three-form $H = dB - \frac{1}{2} \delta_{\alpha\beta} A^\alpha \wedge dA^\beta$ and the abelian two-form field strength $G^\alpha = dA^\alpha$. As derived in detail in [92], the field redefinition is completely given by raising and lowering the indices from the action of the anchor mapping (here $\rho^t = \gamma^{-1}$). For the metric we found in (4.2.11), the quantities in the gravitational sector transform as

$$\begin{aligned} \hat{R}^q_{mnp} &= (\rho^{-1})^q_l \rho^i_m \rho^j_n \rho^k_p R^l_{ijk}, & \hat{R}_{mn} &= \rho^i_m \rho^j_n R_{ij}, \\ \hat{R} &= R, & \sqrt{|\hat{g}|} &= \sqrt{|g|} |\rho^t|, & \hat{\phi} &= \phi \end{aligned} \quad (4.2.27)$$

where the derivative of the transformed theory is (4.2.22).

Concerning the flux sector, so far we know the transformation behavior of the gauge potentials A and B for heterotic frame. One still needs to find the definition of the new field strengths so that they also transform properly, i.e. by raising or lowering indices with the anchor. For that purpose, one needs to invoke the Lie algebroid differential d_E defined in appendix 4.2.1. For the gauge field strength $G = dA$, using the relation (4.2.4) one can show

$$(\Lambda^2 \rho^*) d_E \hat{\mathfrak{A}} = d(\rho^* \hat{\mathfrak{A}}) = dA \quad (4.2.28)$$

with $\rho^* = (\rho^t)^{-1} = \gamma$, so that

$$\hat{\mathfrak{G}} := d_E \hat{\mathfrak{A}} = (\Lambda^2 \rho^t) G \quad (4.2.29)$$

is the correct definition of the transformed field strength. Analogously, one can show

$$d_E \hat{\mathfrak{B}} = (\Lambda^3 \rho^t) dB \quad (4.2.30)$$

so that the proper three-form flux is given by

$$\hat{\mathfrak{H}} := d_E \hat{\mathfrak{B}} - \frac{1}{2} \hat{\mathfrak{A}} \wedge d_E \hat{\mathfrak{A}} = (\Lambda^3 \rho^t) H. \quad (4.2.31)$$

Its Bianchi identity reads

$$d_E \hat{\mathfrak{H}} = -\frac{1}{2} \hat{\mathfrak{G}} \wedge \hat{\mathfrak{G}}. \quad (4.2.32)$$

Thus, each quantity appearing in the heterotic action (4.2.26) now transforms properly so that the action in the redefined fields can be expressed as

$$\mathcal{S} = \int dx \sqrt{\hat{g}} |\rho^*| e^{-2\phi} \left(\hat{R} + 4(D\phi)^2 - \frac{1}{12} \hat{\mathfrak{H}}^{ijk} \hat{\mathfrak{H}}_{ijk} - \frac{1}{4} \hat{\mathfrak{G}}^{ij\alpha} \hat{\mathfrak{G}}_{ij\alpha} \right). \quad (4.2.33)$$

This has an analogous form as the original action, but with the new fields defined in the framework of the differential geometry of the Lie algebroid. Therefore, the latter provides the organizing principle for expressing the action in $O(D, D+n)$ induced redefined field variables.

Note that the symmetries of this action are just the transformed diffeomorphisms, as well as B - and A -field gauge transformations of the original action. Clearly, just by a field redefinition, one does not gain new symmetries. Therefore, the \tilde{A} field gauge transformation (4.1.16), needed for the transition function of the T-dual non-geometric J -flux background is *not* a symmetry of (4.2.33). Thus, as in generalized geometry [92], a field redefinition helps to bring, in each patch, a non-geometric background back to a simple form. However it needs to be emphasized that in general this does not provide a global description of the background, which is distinguished from heterotic DFT framework.

4.2.4 The non-geometric frame

In this section, we show that the field redefinitions between the geometric and the non-geometric frames in section 4.1.1 can also be described in generalized geometry. For that purpose, first recall the form of the generalized metric in these two frames. In the geometric one, we have

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} C_{kj} & -g^{ik} A_{k\beta} \\ -g^{jk} C_{ki} & g_{ij} + C_{ki} g^{kl} C_{lj} + A_i^\gamma A_{j\gamma} & C_{ki} g^{kl} A_{l\beta} + A_{i\beta} \\ -g^{jk} A_{k\alpha} & C_{kj} g^{kl} A_{l\alpha} + A_{j\alpha} & \delta_{\alpha\beta} + A_{k\alpha} g^{kl} A_{l\beta} \end{pmatrix} \quad (4.2.34)$$

and in the non-geometric one

$$\mathcal{H}_{MN} = \begin{pmatrix} \tilde{g}^{ij} + \tilde{C}^{ki} \tilde{g}_{kl} \tilde{C}^{lj} + \tilde{A}^i_\gamma \tilde{A}^{j\gamma} & -\tilde{g}_{jk} \tilde{C}^{ki} & \tilde{C}^{ki} \tilde{g}_{kl} \tilde{A}^l_\beta + \tilde{A}^i_\beta \\ -\tilde{g}_{ik} \tilde{C}^{kj} & \tilde{g}_{ij} & -\tilde{g}_{ik} \tilde{A}^k_\beta \\ \tilde{C}^{kj} \tilde{g}_{kl} \tilde{A}^l_\alpha + \tilde{A}^j_\alpha & -\tilde{g}_{jk} \tilde{A}^k_\alpha & \delta_{\alpha\beta} + \tilde{A}^k_\alpha \tilde{g}_{kl} \tilde{A}^l_\beta \end{pmatrix}. \quad (4.2.35)$$

By comparison of the components, the corresponding field redefinition takes the form

$$\begin{aligned}\tilde{g} &= g + C^t g^{-1} C + A^2 \\ \tilde{C} &= \tilde{g}^{-1} C^t g^{-1} \\ \tilde{A} &= -(\tilde{g}^{-1} + \tilde{C}) A.\end{aligned}\tag{4.2.36}$$

Analogous to [92], we propose that this transformation is implemented by choosing

$$\mathcal{M} = \begin{pmatrix} 0 & \tilde{g} & 0 \\ \tilde{g}^{-1} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\tag{4.2.37}$$

with $\tilde{g} = g + C^t g^{-1} C + A^2$. Evaluating the expressions (4.2.12), (4.2.14), (4.2.15), (4.2.18) we obtain as intermediate results

$$\begin{aligned}\gamma &= -(g + C) \tilde{g}^{-1} & \text{so that } \gamma^{-1} &= -(g + C^t) g^{-1}, \\ \delta &= -\tilde{g} & \text{so that } \mathfrak{C} &= C^t, \\ \mathfrak{A} &= A.\end{aligned}\tag{4.2.38}$$

Using these relations further in (4.2.11), (4.2.14) and (4.2.17) we finally get

$$\begin{aligned}\hat{g} &= (\gamma^{-1}) g (\gamma^{-1})^t = \tilde{g} \\ \hat{C} &= (\gamma^{-1}) \mathfrak{C} (\gamma^{-1})^t = C^t g^{-1} \tilde{g} \\ \hat{A} &= (\gamma^{-1}) \mathfrak{A} = -(1 + C^t g^{-1}) A.\end{aligned}\tag{4.2.39}$$

Here \hat{C} and \hat{A} are still forms. To transform them into a bi-vector and a vector, one raises the indices with \tilde{g}^{-1} so that

$$\begin{aligned}\tilde{C} &= \tilde{g}^{-1} \hat{C} \tilde{g}^{-1} = \tilde{g}^{-1} C^t g^{-1} \\ \tilde{A} &= \tilde{g}^{-1} \hat{A} = -(\tilde{g}^{-1} + \tilde{C}) A,\end{aligned}\tag{4.2.40}$$

which precisely agrees with the field redefinition of the non-geometric frame (4.2.36) in heterotic DFT.

4.3 Lie algebroid and $SL(5)$ transformation

In this section we review the generalized geometry for M-theory on an orientable n -dimensional manifold \mathcal{M} [145]. In this frame the generalized bundle $TM \oplus T^*M$ with a natural action of $O(D, D)$ is generalized by $TM \oplus \Lambda^2 T^*M \oplus \dots$ with a natural action of E_n , and the 2-form symmetry of B-shifts is generalized to 3-form shifts.

For $E_4 = SL(5, R)$, we consider a four dimensional manifold. The bundle is $E = TM \oplus \Lambda^2 T^*M$ with 10-dimensional fibers transforming in the $\mathbf{4} + \mathbf{6}$ representation of $SL(4)$. The section is a formal sum $U \equiv \nu + \rho$ of a vector ν and a 2-form ρ which can be thought of as an extended vector with 10 components $U^I (I = 1, \dots, 10)$

$$U^I = \begin{pmatrix} \nu^i \\ \rho_{ij} \end{pmatrix}. \quad (4.3.1)$$

where $i, j = 1, \dots, 4$ and $\rho_{ij} = -\rho_{ji}$.

To study about the $SL(5)$ transformation, we use the convenient little metric m_{ab} (2.4.9). The general $SL(5)$ transformation matrix takes the form

$$U_a{}^b = \begin{pmatrix} a_i{}^j & b_i \\ c^j & d \end{pmatrix}. \quad (4.3.2)$$

The generalized metric transform as $\tilde{m}_{ab} = U_a{}^c m_{cd} U^d{}_b$. Reading off the upper-left component of \tilde{m}_{ab} we have in the geometric frame

$$e^{-\tilde{\phi}/2} |\tilde{g}|^{-1/2} \tilde{g} = e^{-\phi/2} [|g|^{-1/2} a g a^t + b(aV)^t + (aV)b^t + |g|^{1/2} (1 + V^2) b b^t], \quad (4.3.3)$$

with $V_i = V, V_j = V^t$. From the density invariance under local Riemannian geometry $SO(5)$ [146], which is equivalent to the determinant of the generalized metric to be invariant, we have the relation

$$e^{-5\phi/2} g^{-1/2} = e^{-5\tilde{\phi}/2} \tilde{g}^{-1/2}. \quad (4.3.4)$$

Recall that the warp factor $e^\phi \equiv |g_7|^{1/7}$, which can be redefined according to the external metric g_{IJ} (in the transverse seven directions). Namely, when we define

$$\tilde{g}_{IJ} = X(g_{ij}, V_i) g_{IJ} \quad (4.3.5)$$

we have

$$e^{5\phi/2} e^{-5\tilde{\phi}/2} = X^{-5/2}. \quad (4.3.6)$$

Following from (2.4.9), (4.3.3) and (4.3.5), the metric \tilde{g} in the new frame can be expressed

as

$$\tilde{g} = X^{-2}[(a + |g|^{1/2}bV^t)g(a^t + |g|^{1/2}Vb^t) + |g|bb^t]. \quad (4.3.7)$$

By defining the matrix γ (which is the transverse of the anchor for the lie-Algebroids as we will discuss later) in the form of

$$\gamma = a + |g|^{1/2}bV^t \quad (4.3.8)$$

we have \tilde{g}_{ij} simplified as

$$\tilde{g} = X^{-2}\left(\gamma[g + (\gamma^{-1}b)|g|(\gamma^{-1}b)^t]\gamma^t\right). \quad (4.3.9)$$

Similarly, we have the transformed vector in the new frame as

$$\tilde{V} = X^{1/2}\left(\gamma[Vd^t + |g|^{1/2}(gc^t + \gamma^{-1}bd^t)]\right). \quad (4.3.10)$$

Now we have the candidate for the anchor of $SL(5)$ Lie-Algebroids as

$$\rho = \gamma^t. \quad (4.3.11)$$

Note that the $SL(5)$ theory presents a toy model of M-theory with seven external dimensions. When we choose U_a^b to be the identity matrix and $X(g_{ij}, V_i) = (1 + V^2)^{-1/3}$, the generalized metric in the new frame precisely recovers it in the non-geometric frame of $SL(5)$ theory (2.4.10) under $SL(5)$ transformation $\tilde{m}_{ab} = U_a^c m_{cd} U^d_b$. Moreover, the geometric and non-geometric fields are related to each other through the field redefinition given in [127]

$$\begin{aligned} \tilde{g}_{ij} &= (1 + V^2)^{-1/3}((1 + V^2)g_{ij} - V_i V_j), \\ \Omega^{ijk} &= (1 + V^2)^{-1}g^{il}g^{jm}g^{kn}C_{kmn}, \\ \tilde{g}_{IJ} &= (1 + V^2)^{-1/3}g_{IJ}. \end{aligned} \quad (4.3.12)$$

4.4 Summary and discussion

In this chapter, we studied a couple of aspects of heterotic DFT in more detail. We think that, while the general formalism of heterotic DFT was developed before and is a straightforward generalization of bosonic DFT, the concrete evaluation of its consequences, in particular for issues related to the gauge field, deserved a further study.

Indeed, by applying the T-duality rules (α' corrected heterotic Buscher rules) to a flat background with a constant gauge field, we found non-geometric backgrounds, which were very similar to the Q - and R -flux backgrounds in bosonic DFT. Namely, after one

T-duality we already obtained a background which was best described by changing to a non-geometric frame, where the gauge one-form has turned into a gauge one-vector. The required transition function between two patches was given by a new symmetry, namely a one-vector gauge transformation involving a winding dependence. Thus, this background is globally non-geometric, an effect introduced by the α' corrected Buscher rules. Applying a further T-duality, the arising background was even locally non-geometric.

Even though we were only considering abelian gauge fields, we expect this picture to generalize also to non-abelian gauge fields. The latter are introduced via a gauging procedure that generically breaks the $O(D, D + n)$ symmetry to $O(D, D)$. However, T-duality is a special element of $O(D, D)$ so that it can still be treated analogously to the abelian case.

Moreover, we clarified which type of fluxes are turned on in these backgrounds and how they are microscopically described in terms of the fundamental fields in the theory. We argued that the constant non-geometric J -flux background of the $E_8 \times E_8$ heterotic string can be considered the S-dual of a type I' background with a $D8$ -brane intersecting the $O8$ -plane at an angle.

Led by the apparent necessity of field redefinitions, we considered the general question what effect an $O(D, D + n)$ induced field redefinition has on the heterotic supergravity action. Generalizing [92], we investigated this question in the framework of generalized geometry and found very similar results, though now including various corrections due to the present one-form gauge field. In particular, the organizing principle for the terms in the redefined action was given by the differential geometry of a Lie algebroid, whose anchor was related to the $O(D, D + n)$ transformation. The non-geometric frame was identified with just a specific $O(D, D + n)$ induced field redefinition. Furthermore, we generalized this study to $SL(5)$ theory.

Even though here we were only considering the NS part of the heterotic action, we expect that the whole action including the fermionic terms are governed by the objects in the differential geometry of the Lie algebroid. This includes e.g. the kinetic terms for the gravitinos and gluinos, that involve a spin-connection. Moreover, here we were neglecting the gravitational Chern-Simons term (see [101] for a treatment in DFT). Introducing non-abelian gauge fields via gauging, breaks the $O(D, D + n)$ symmetry so that in this case only the remaining symmetry $O(D, D)$ should be used for a field redefinition.

Non-associative Deformations of Geometry

In this chapter, we will study the application of (non-)geometric flux backgrounds in deformations of geometry. Recall that the basic starting point of string theory is the two-dimensional field theory on the world-volume of the probe string equipped with the fundamental paradigm that on-shell solutions of string theory are provided by two-dimensional Conformal Field Theories (CFTs) with critical central charges. However, the generic left-right asymmetric CFT does not correspond to a fixed point of a non-linear sigma model with a geometric target space. Incorporating T-duality, a natural question to ask is whether there is a proper non-geometric generalization that the generic asymmetric CFT corresponds to. Since string theory is strongly believed to provide a consistent theory of quantum gravity, this could also shed light on the study of quantum gravity and non-commutative geometry. In the last years, the development of Double Field Theory (DFT) and generalized geometry has accelerated the study on a better understanding of the non-geometric regime in string theory. Some recent developments go precisely in the direction of providing a quasi-geometric description of these asymmetric CFTs, in which T-duality played an important role.

Recall that the simple closed string background of a flat space with constant H -flux and dilaton was considered in [61]. Successively applying the Buscher rules, one arrives at the well-known T-dual background flux chain $H_{ijk} \xleftarrow{T_k} F_{ij}{}^k \xleftarrow{T_j} Q_i{}^{jk} \xleftarrow{T_i} R^{ijk}$. The last two flux backgrounds were argued to be non-geometric. The Q -flux case is still locally geometric but the transition functions involve non-geometric T-duality transformations, while the R -flux case is considered to be also locally non-geometric. Furthermore, it is worthwhile to point out that this background does not allow the notion of a point [147]. Consider a D3-brane wrapping a three-torus carrying a constant three-form H -flux. In fact such a configuration is not allowed as it suffers from the Freed-Witten anomaly [148], i.e. it violates the Bianchi identity $d\mathcal{F} = H$ for the gauge flux on the brane. However by formally applying successive T-dualities along all three directions of the torus, one gets a D0-brane with transverse R -flux. Thus, placing a point-like object in an R -flux background is not allowed. This suggests an uncertainty relation $\Delta x \Delta y \Delta z \geq \ell_s^4 R^{xyz}$ which indicates a relation to non-commutative geometry. Indeed, it was briefly argued in [149] that the R -flux involves a non-associativity of the coordinates. For closed strings that

are winding and moving in non-geometric backgrounds, it was found in [112, 113, 150–152] that by explicit string and CFT computations, the string geometry indeed becomes non-commutative and non-associative. Concretely, the equal-time cyclic double-commutator of three local coordinates was shown to be

$$[x^i, x^j, x^k] = \begin{cases} 0 & H\text{-flux} \\ \ell_s^4 R^{ijk} & R\text{-flux} \end{cases} . \quad (5.0.1)$$

Similar results arise for a commutator algebra

$$[x^i, x^j] = \frac{i}{3\hbar} \ell_s^4 R^{ijk} p_k, \quad [x^i, p_j] = i\hbar \delta^i_j, \quad (5.0.2)$$

so that the Jacobiator gives precisely (5.0.1). If Q -flux is also present, the commutator was generalized to

$$[x^i, x^j] = \frac{i}{3\hbar} \ell_s^4 \left(R^{ijk} p_k + Q_k^{ij} w^k \right), \quad (5.0.3)$$

where w^k is the winding operator. Analogous relations were derived in the framework of matrix theory in [153] as well.

In [113] this (non-)geometry background was investigated using conformal perturbation theory, and analogous to the open string story [108], on-shell string scattering amplitudes of tachyons were computed. For both constant H -flux and R -flux the final scattering amplitudes were associative, as expected from crossing symmetry of conformal correlation functions. However, prior to invoking momentum conservation, there was a difference between the H - and R -flux case. For the H -flux the holomorphic and anti-holomorphic phases directly canceled each other while for the R -flux they added up. These phases could be encoded (at least at linear order in R^{ijk}) in the tri-product¹

$$(f \triangle g \triangle h)(x) = \exp\left(\frac{\ell_s^4}{6} R^{ijk} \partial_i^{x_1} \partial_j^{x_2} \partial_k^{x_3}\right) f(x_1) g(x_2) h(x_3) \Big|_x. \quad (5.0.5)$$

The three-bracket can then be defined as

$$[x^i, x^j, x^k] = \sum_{\sigma \in S_3} \text{sign}(\sigma) x^{\sigma(i)} \triangle x^{\sigma(j)} \triangle x^{\sigma(k)}, \quad (5.0.6)$$

where S_3 denotes the permutation group of three elements. Note that, formally one can

¹Choosing $f = \exp(ip_1 x)$ and similar for g, h the momentum conservation can be implemented by integrating the tri-product (5.0.5), so that the order ℓ_s^4 correction becomes

$$\int d^n x R^{ijk} p_i^1 p_j^2 p_k^3 e^{i(p^1 + p^2 + p^3) \cdot x} = R^{ijk} p_i^1 p_j^2 p_k^3 \delta(p^1 + p^2 + p^3) = 0. \quad (5.0.4)$$

The aim of this chapter is to generalize this result to non-constant fluxes on a curved space.

also define such tri-product with H^{ijk} instead of R^{ijk} . By starting with the non-associative commutator algebra (5.0.2), the tri-product (5.0.5) as well as an associated momentum dependent star-product were derived in [154, 155]. In addition, the non-commutative and non-associative phase space structure of DFT as well as the magnetic field analogue to the string R -flux model were discussed in [155].

Recall that the DFT framework provides an $O(D, D)$ covariant formulation of the massless modes of string theory. This was initiated in [78, 79] and pushed forward more recently in [81–83, 156] (see [63, 84, 157] for reviews). It doubles the number of target space coordinates by also introducing winding coordinates. It turned out that this is a constrained theory, since usually the weak and the strong constraint are imposed. Locally one ends up on a D -dimensional slice of the $2D$ -dimensional doubled geometry, which can be rotated to the supergravity frame via an $O(D, D)$ transformation. Moreover, it admits all the local symmetries, usual and winding diffeomorphisms, to allow for a global description of Q - and R -flux backgrounds. This is possible as T-duality exchanges ordinary and winding coordinates so that for these non-geometric backgrounds there appears a winding coordinate dependence either in the transition functions between two charts (Q -flux) or in the definition of the flux itself (R -flux). Thus, non-geometry just means explicit winding coordinate dependence in the background fluxes or in the transition functions. Furthermore, it is shown in [85–87], that DFT is also related to generalized geometry by annihilating the winding coordinates and constructing the generalized tangent bundle as $TM \oplus T^*M$ with only usual coordinates.

There exist two kinds of formulations of DFT. First, there is the generalized metric formulation, which was developed in a series of papers [81–83, 156]. Here one imposes the so-called strong constraint to guarantee e.g. closure of the symmetry algebra (the C-bracket). Based on the previous work [78–80] and [94, 100, 122, 123], in [66] a flux formulation of DFT has been provided which incorporates the relation to gauged supergravity theories. It was shown that the flux formulation is equivalent to the generalized metric formulation, up to boundary terms and terms vanishing by the strong constraint. It allows to move away from the strong constraint and admit truly non-geometric duality orbits of fluxes in the sense of [158]. In fact, it makes use of the observation that requiring only closure of the symmetry algebra provides a (weaker) closure constraint than the strong constraint. A weakening of the strong constraint was first discussed in [159]. Some simple examples were analyzed via by Scherk-Schwarz reductions [160, 161] of DFT [66, 100, 122, 123] (see also [162, 163]). Note that in [164] concrete examples of asymmetric orbifold CFTs were presented. It was shown that they do correspond to non-geometric duality orbits.

It was shown in [165] that DFT originated as a background independent formalism, where the generalized coordinate transformations compose in a non-standard manner, such that the geometry is non-associative. However this non-associativity vanishes after imposing the strong constraint on arbitrary fields. It is interesting to study the non-associative deformation via the tri-product (5.0.5), in the DFT framework with R -flux present. This is the essential aim of this chapter. We will also focus on the flux formulations of DFT. To

start our discussion, we identify two important aspects:

- First, (5.0.5) implies that non-associativity arises for an R -flux background contracted with ordinary partial derivatives $\partial/\partial x^i$. Note that, in this sense the DFT T-dual of the H -flux background on ordinary space is an R -flux background on winding space.
- Second, in quantum theories, where observables are operators acting on some Hilbert space, one can get non-commutativity, but the product of operators is always associative. Since CFTs are ordinary (2-dimensional) quantum theories, on-shell, i.e. if the string equations of motion are satisfied, there should better not be any violation of associativity in CFT on-shell scattering amplitudes.

In CFT one requires crossing symmetry of the operator product expansion, which is related to the Jacobi identities for the algebra of the modes of the conformal fields. In string theory, from on-shell scattering amplitudes one can determine an effective theory for the massless modes (as in DFT frameworks), which by construction does not exhibit non-associativity on shell. We will show that any admissible non-associative deformation given by a non-associative tri-product like (5.0.5) has a trivial effect on the effective field theory, when going on-shell. However it is not clear whether the off-shell effective string action is sensitive against non-associative deformations of the underlying geometry. As we will discuss, the main result of this chapter is that, on the level of the effective action, a non-associative deformation of the DFT generalization of both the H -flux and the R -flux leads at most to boundary terms. For the H -flux case we invoke the DFT equations of motion, whereas for the second deformation with R -flux, it turns out to be trivial once one imposes either the strong or even the closure constraint.

A similar reasoning also applies to the case of open strings ending on D-branes supporting a non-trivial, in general non-constant gauge flux. The case when this product becomes non-associative was analyzed in a series of papers [109, 110, 166]. Thus, before we move on to briefly review the flux formulation of DFT in section 2.3, we present in section 5.1 two known examples of non-associativity, namely the system of an electric charge moving in a magnetic monopole field and a D-brane carrying non-constant gauge flux. In section 5.3 we will analyze possible tri-products for DFT. As we will see, there are two candidates, one related to the tri-product (5.0.5) with H -flux and one to the tri-product with R -flux. Both cases will be discussed in detail. This work will also be extended to a deformations of geometry study in heterotic DFT framework, and the results are summarized in [102].

5.1 Non-associativity in physics

In this section we review two instances where a non-associative structure has appeared in physics. First, we recall the story of quantizing the motion of an electrically charged particle in a magnetic field. Second, the effective theory on a D-brane with non-constant

magnetic background field turned on is considered. This gives a non-vanishing $H = dB$ flux, which in general leads to a non-associative star-product.

5.1.1 Non-associativity for magnetic monopoles

As it is known for some time [167–171], non-associativity emerges when one considers the quantization of a charged particle in the background of a magnetic monopole. Furthermore, it is shown in [155] that how non-associativity of geometry can be reconciled, while as in quantum mechanics it is required that all the operators are associative. At first, we recall a few facts about this system following essentially [167, 168]. The commutator algebra between position and momentum of a particle in a background magnetic field \vec{B} in three space-dimensions takes the following form

$$[x^i, p_j] = i\hbar\delta_j^i, \quad [x^i, x^j] = 0, \quad [p_i, p_j] = i\hbar e \epsilon^{ijk} B_k(\vec{x}). \quad (5.1.1)$$

In turn, the Jacobiator becomes

$$[p_i, p_j, p_k] = -e\hbar^2 \epsilon^{ijk} \vec{\nabla} \cdot \vec{B} \quad (5.1.2)$$

with $\vec{\nabla} \cdot \vec{B} = 4\pi\rho_m$ in Gaussian-cgs units. These relations take analogous forms as the commutators (5.0.2) and three-bracket (5.0.1) after exchanging the role of momentum and position variables in these equations.

Now, we consider the finite translation operators $U(a) = \exp(\frac{i}{\hbar}a \cdot p)$. Using the Baker-Campbell-Hausdorff formula one obtains

$$U(a)U(b) = \exp\left(-\frac{ie}{\hbar}\Phi_{(a,b)}\right)U(a+b) \quad (5.1.3)$$

where $\Phi_{(a,b)} = \frac{1}{2}(a \times b)^k B_k$ denotes the magnetic flux through the (infinitesimally small) triangle spanned by the two vectors (a, b) . Similarly, one can compute the associator of three U s

$$(U(a)U(b))U(c) = \exp\left(-\frac{ie}{\hbar}\Phi_{(a,b,c)}\right)U(a)(U(b)U(c)) \quad (5.1.4)$$

where $\Phi_{(a,b,c)} = \frac{1}{6}[(a \times b) \cdot c] \vec{\nabla} \cdot \vec{B}$ denotes the magnetic flux through the tetrahedron spanned by the three vectors (a, b, c) . Due to Gauss law, this is precisely the magnetic charge $4\pi m$ sitting inside of the tetrahedron. Therefore, the non-associativity (5.1.4) vanishes if

$$\frac{em}{\hbar} = \frac{N}{2} \quad (5.1.5)$$

with an integer N . This is Dirac's quantization rule for the magnetic charge as addressed in [167].

In this chapter, we are essentially generalizing the above mentioned ideas to DFT framework and study the non-associativity with non-geometric backgrounds incorporated. We require that the non-associative tri-product deformation of the DFT action to be consistent with the requirements from CFT scattering amplitudes. The main difference is that we are not considering quantized fluxes and momenta but the case where these are in general non-rational and spacetime dependent. From the requirement of absence of non-associativity we aim to learn something fundamental about the system.

5.1.2 Open string with non-associative star product

Recall that the CFT of an open string ending on a D-brane supporting a non-trivial gauge flux $\mathcal{F} = B + 2\pi\alpha' F$ features a non-commutative geometry.

By computing the disc level scattering amplitude of N -tachyons, certain relative phases appear, which for constant gauge flux can be described by the Moyal-Weyl star-product

$$(f \star g)(x) = \exp\left(i \frac{\ell_s^2}{2} \theta^{ij} \partial_i^{x_1} \partial_j^{x_2}\right) f(x_1) g(x_2) \Big|_x, \quad (5.1.6)$$

where the relation of the open and closed string quantities is

$$G^{-1} + \theta = (g + \mathcal{F})^{-1}. \quad (5.1.7)$$

In the Seiberg-Witten limit, the OPE becomes exactly the Moyal-Weyl star-product. This non-trivial product of functions lead to the non-commutative Moyal-Weyl plane with $[x^i, x^j] = i \ell_s^2 \theta^{ij}$. For on-shell string scattering amplitudes such a non-commutativity can show up, because the conformal $SL(2, \mathbb{R})$ symmetry group only leaves the cyclic order of the inserted vertex operators invariant. By the same reason, non-commutativity must preserve cyclicity on shell.

It has been shown in [172] that for every Poisson structure θ^{ij} one can define a corresponding associative star-product, which will also involve derivatives of the Poisson structure. The same product can also be considered for a quasi Poisson structure, but leads to a non-associative star-product. This is related to the physical situation of an open string ending on a D-brane with generic non-constant B -field, i.e. non-vanishing field strength H . At leading order in derivatives this leads to a non-commutative product

$$\begin{aligned} f \circ g = f \cdot g + & i \frac{\ell_s^2}{2} \theta^{ij} \partial_i f \partial_j g - \frac{\ell_s^4}{8} \theta^{ij} \theta^{kl} \partial_i \partial_k f \partial_j \partial_l g \\ & - \frac{\ell_s^4}{12} (\theta^{im} \partial_m \theta^{jk}) (\partial_i \partial_j f \partial_k g + \partial_i \partial_j g \partial_k f) \dots \end{aligned} \quad (5.1.8)$$

The associator for this product becomes

$$(f \circ g) \circ h - f \circ (g \circ h) = \frac{\ell_s^4}{6} \theta^{ijk} \partial_i f \partial_j g \partial_k h + \dots \quad (5.1.9)$$

with $\theta^{ijk} = 3 \theta^{[im} \partial_m \theta^{jk]}$, which precisely vanishes for a Poisson tensor.

For open string, such a non-associative deformation of the underlying spacetime has been analyzed in [109, 166]. From open string scattering amplitudes one can determine the low-energy effective action so that also the effect of non-associativity in its quantum deformation should be trivial. Indeed, consider the DBI action

$$S_{\text{DBI}} = \int d^n x \sqrt{g + \mathcal{F}} \quad (5.1.10)$$

and vary it with respect to the gauge potential A in $\mathcal{F} = B + dA$. One gets

$$\partial_i \left(\sqrt{g + \mathcal{F}} [(g + \mathcal{F})^{-1}]^{[ij]} \right) = \partial_i \left(\sqrt{g + \mathcal{F}} \theta^{ij} \right) = 0 \quad (5.1.11)$$

where we have used (5.1.7). Then, it directly follows that up to leading order in $\partial\theta$ the \star -product satisfies the property

$$\int d^n x \sqrt{g + \mathcal{F}} f \circ g = \int d^n x \sqrt{g + \mathcal{F}} f \cdot g. \quad (5.1.12)$$

Indeed, e.g. at order $O(\ell_s^2)$ the difference between the left and the right hand side is a total derivative on-shell

$$i \frac{\ell_s^2}{2} \int d^n x \sqrt{g + \mathcal{F}} \theta^{ij} \partial_i f \partial_j g = i \frac{\ell_s^2}{2} \int d^n x \partial_i \left(\sqrt{g + \mathcal{F}} \theta^{ij} f \partial_j g \right) = 0 \quad (5.1.13)$$

where here and in the following sections we always assume that the functions f, g are sufficiently well behaving, so that integrals over total derivatives vanish. Thus, as expected from CFT, in the effective action the product of two functions is commutative (cyclic), once the background satisfies the string equations of motion.

Similarly, the associator below the integral also gives a total derivative at leading order in $\partial\theta$. E.g. at order $O(\ell_s^4)$ we find

$$\int d^n x \sqrt{g + \mathcal{F}} \left((f \circ g) \circ h - f \circ (g \circ h) \right) = \frac{\ell_s^4}{6} \int d^n x \partial_i \left(\sqrt{g + \mathcal{F}} \theta^{ijk} f \partial_j g \partial_k h \right) = 0, \quad (5.1.14)$$

where we have used

$$\partial_i \left(\sqrt{g + \mathcal{F}} \theta^{ijk} \right) = 0, \quad (5.1.15)$$

which can be seen by expanding θ^{ijk} and successively employing the equation of motion (5.1.11) and the anti-symmetry of θ^{ij} . The two relations (5.1.12) and (5.1.14) also hold for higher orders in derivatives of θ^{ij} [166]. Note that, as one is using the DBI action, the star-product is exact in α' at leading order in $\partial\theta$. Thus, we conclude that on-shell the non-associativity of the \circ -product is not visible, as expected from the open string CFT.

In the following we will generalize this kind of analysis to the closed string case. Since there we are dealing with non-geometric fluxes, the appropriate framework is DFT. Therefore, let us recall those aspects of DFT which will be used in the main section 5.3.

5.2 Flux formulation of DFT

The explicit form of the geometric and non-geometric fluxes H, F, Q and R in terms of B and β can be computed according to (2.3.21, 2.3.22). The explicit form of the fluxes were shown in [66, 67, 89, 122]. For later discussion, we list the fluxes in the non-geometric backgrounds with $B_{mn} = 0$ in (2.3.19). Defining

$$f^c{}_{ab} = e_i{}^c \left(\partial_a e_b{}^i - \partial_b e_a{}^i \right), \quad \tilde{f}_a{}^{bc} = e_a{}^i \left(\tilde{\partial}^b e_i{}^c - \tilde{\partial}^c e_i{}^b \right), \quad (5.2.1)$$

one finds $H_{abc} = 0$ and the geometric flux $F^c{}_{ab} = f^c{}_{ab}$. The non-geometric fluxes are

$$Q_c{}^{ab} = \tilde{f}_c{}^{ab} + \partial_c \beta^{ab} + f^a{}_{cm} \beta^{mb} + f^b{}_{cm} \beta^{am} \quad (5.2.2)$$

and ²

$$R^{abc} = 3 \left(\tilde{\partial}^{[a} \beta^{bc]} + \tilde{f}_m{}^{[ab} \beta^{c]m} \right) + 3 \left(\beta^{[am} \partial_m \beta^{bc]} + \beta^{[am} \beta^{bn} f^c{}_{mn} \right). \quad (5.2.3)$$

For later discussion, we analyze some of the consequences by imposing the closure constraint from DFT. At first, if f is a generalized scalar, we can write

$$\mathcal{D}_A f = E_A{}^M \partial_M f = \mathcal{L}_{E_A}(f), \quad (5.2.4)$$

which by the closure constraint implies that $\Delta_\xi(\mathcal{L}_{E_A} f) = 0$. Therefore, $\mathcal{D}_A f$ is also a generalized scalar. Now, by direct computation one obtains

$$\Delta_\xi(\mathcal{D}_B f) = \delta_\xi(\mathcal{D}_B f) - \mathcal{L}_\xi(\mathcal{D}_B f) = (\mathcal{D}^C \xi^M) E_{BM} \mathcal{D}_C f = 0. \quad (5.2.5)$$

Thus, choosing $\xi = E_A$ we can conclude

$$(\mathcal{D}^C E_A{}^M) E_{BM} \mathcal{D}_C f = \Omega^C{}_{AB} \mathcal{D}_C f = 0. \quad (5.2.6)$$

²Similar to the open string case (5.1.9), the contribution $R_{cl}^{abc} = 3(\beta^{[am} \partial_m \beta^{bc]} + \dots)$ can be considered as the defect for associativity, when we consider β^{ab} as a classical (quasi-) Poisson tensor.

For a generalized scalar g , we can also choose $\xi = E_B g$ in (5.2.5) and obtain using the relation (5.2.6)

$$\delta_{AB} \mathcal{D}^C g \mathcal{D}_C f = 0. \quad (5.2.7)$$

Hence, we conclude that the closure constraint implies that for scalars f and g the strong constraint still has to hold. A particular example which we will use later is

$$(\mathcal{D}_C \mathcal{F}_A) \mathcal{D}^C f = 0. \quad (5.2.8)$$

Similarly, the fluxes $\mathcal{F}_{ABC} = E_{CM} (\mathcal{L}_{E_A} E_B^M)$ and $\mathcal{F}_A = -e^{2d} (\mathcal{L}_{E_A} e^{-2d})$ with flat indices transform as scalars with respect to generalized diffeomorphisms, i.e.

$$\delta_\xi \mathcal{F}_{ABC} = \xi^M \partial_M \mathcal{F}_{ABC}, \quad \delta_\xi \mathcal{F}_A = \xi^M \partial_M \mathcal{F}_A. \quad (5.2.9)$$

However, under a local double Lorentz transformation one gets

$$\delta_\Lambda \mathcal{F}_{ABC} = 3 \left[\mathcal{D}_{[A} \Lambda_{BC]} + \Lambda_{[A}{}^D \mathcal{F}_{BC]D} \right], \quad \delta_\Lambda \mathcal{F}_A = \mathcal{D}^B \Lambda_{BA} + \Lambda_A{}^B \mathcal{F}_B, \quad (5.2.10)$$

where the first terms are anomalous. One often write e.g. $\Delta_\Lambda \mathcal{F}_{ABC} = 3 \mathcal{D}_{[A} \Lambda_{BC]}$. For the relation (5.2.6) to be well defined we require

$$0 = \Delta_\Lambda (\Omega^C{}_{AB} \mathcal{D}_C f) = (\mathcal{D}^C \Lambda_{AB}) \mathcal{D}_C f, \quad (5.2.11)$$

which could also be read off from (5.2.7). Moreover, the fluxes satisfy the generalized Bianchi identities

$$\mathcal{D}_{[A} \mathcal{F}_{BCD]} - \frac{3}{4} \mathcal{F}_{[AB}{}^M \mathcal{F}_{CD]M} = \mathcal{Z}_{ABCD} \quad (5.2.12)$$

and

$$\mathcal{D}^M \mathcal{F}_{MAB} + 2 \mathcal{D}_{[A} \mathcal{F}_{B]} - \mathcal{F}^M \mathcal{F}_{MAB} = \mathcal{Z}_{AB}, \quad (5.2.13)$$

where the right hand sides are given by

$$\begin{aligned} \mathcal{Z}_{ABCD} &= -\frac{3}{4} \Omega_{E[AB} \Omega^E{}_{CD]} \\ \mathcal{Z}_{AB} &= \left(\partial^M \partial_M E_{[A}{}^N \right) E_{B]N} - 2 \Omega^C{}_{AB} \mathcal{D}_C d. \end{aligned} \quad (5.2.14)$$

Both quantities vanish by the strong constraint. As it is shown in [66], note that $\Delta_{E_A} \mathcal{F}_B = \mathcal{Z}_{AB}$ and $\Delta_{E_A} \mathcal{F}_{BCD} = \mathcal{Z}_{ABCD}$, this also holds for the closure constraint.

Due to (5.2.9) the DFT action (2.3.26) is apparently invariant under generalized diffeomorphisms. Taking the anomalous terms in (5.2.10) into account, under local double

Lorentz transformations, the action transforms into a boundary term plus

$$\delta_\Lambda S_{\text{DFT}} = \int dX e^{-2d} \Lambda_A{}^C (\eta^{AB} - S^{AB}) \mathcal{Z}_{BC} \quad (5.2.15)$$

which indeed vanishes for all possible constraints. The derivative (2.3.20) satisfies the commutation relations

$$[\mathcal{D}_A, \mathcal{D}_B] = \mathcal{F}^C{}_{AB} \mathcal{D}_C - \Omega^C{}_{AB} \mathcal{D}_C = \mathcal{F}^C{}_{AB} \mathcal{D}_C, \quad (5.2.16)$$

where $\Omega^C{}_{AB} \mathcal{D}_C$ vanishes after invoking either the strong or the closure constraint (5.2.6). Varying the action with respect to the vielbeins, one obtains the equations of motion

$$\mathcal{G}^{[AB]} = \mathcal{Z}^{AB} + 2S^{C[A} \mathcal{D}^{B]} \mathcal{F}_C + (\mathcal{F}_C - \mathcal{D}_C) \check{\mathcal{F}}^{C[AB]} + \check{\mathcal{F}}^{CD[A} \mathcal{F}_{CD}{}^{B]} = 0 \quad (5.2.17)$$

with

$$\check{\mathcal{F}}^{ABC} = \check{S}^{ABCDEF} \mathcal{F}_{DEF} \quad (5.2.18)$$

and

$$\check{S}^{ABCDEF} = \frac{1}{2} S^{AD} \eta^{BE} \eta^{CF} + \frac{1}{2} \eta^{AD} S^{BE} \eta^{CF} + \frac{1}{2} \eta^{AD} \eta^{BE} S^{CF} - \frac{1}{2} S^{AD} S^{BE} S^{CF}. \quad (5.2.19)$$

Note that the Ω -odd terms in (2.3.26) do not contribute to these equations of motion. The dilaton equation of motion is that the integrand of the action (2.3.26) vanishes. It is remarkable that it is possible to express the equations of motions, including the gravity part, in this unified way just in terms of doubled fluxes \mathcal{F}_{ABC} and \mathcal{F}_A .

Finally, let us mention that, by analyzing a Scherk-Schwarz reduction of DFT, it was pointed out in [122, 123] that the quadratic constraints of gauged supergravity are satisfied even though the strong constraint is not. Additionally, in [66, 100] it was shown that for such Scherk-Schwarz reductions the closure constraint of DFT is satisfied. Thus, in a compactified DFT the strong constraint seems only to be a sufficient but not a necessary requirement. These Scherk-Schwarz reductions provide explicit examples of truly doubled geometries [158]. However, whether all such non-geometric backgrounds are honest solutions of string theory is still under debate.

5.3 Non-associative deformations of DFT

In this section we investigate the generalization of the open string analysis of section 5.1.2 to the closed string, which we describe by DFT. As we argued (on-shell) closed string scattering amplitudes are not expected to show any sign of non-associativity. The latter

is due to the fact that CFT amplitudes are crossing symmetric, which corresponds to satisfied Jacobi-identities in an operator formalism. Therefore, we again expect that the deformation of the effective action by a (non-associative) tri-product should better be trivial (at least) on-shell. However, let us stress that, if one can identify such a specific non-trivial tri-product, one definitely has made a big change of the underlying geometry. We will show that, under certain conditions, it remarkably has no effect for the DFT action. In a similar vein, the conformal $SL(2, \mathbb{C})$ symmetry does not preserve the (radial) ordering of points on the sphere. Therefore, on-shell one also does not expect to see any imprint of non-commutativity.

In DFT, there exist two possible tri-products. First, there is the tri-product

$$f \triangle g \triangle h = f g h + \frac{\ell_s^4}{6} \check{\mathcal{F}}^{ABC} \mathcal{D}_A f \mathcal{D}_B g \mathcal{D}_C h + O(\ell_s^8). \quad (5.3.1)$$

Since (5.3.1) contains the component $H^{abc} \partial_a f \partial_b g \partial_c h$, with $H^{ijk} = g^{ii'} g^{jj'} g^{kk'} H_{i'j'k'}$, it can be considered as the DFT generalization of the three-product (5.0.5) with H -flux deformation. Even though there does not exist evidence for the presence of some non-associativity for H -flux, we study it here as it is the direct generalization of the open string story and still shows some remarkable properties.

The second possibility is the generalization of the tri-product with R^{ijk} deformation

$$f \triangle g \triangle h = f g h + \frac{\ell_s^4}{6} \mathcal{F}_{ABC} \mathcal{D}^A f \mathcal{D}^B g \mathcal{D}^C h + O(\ell_s^8). \quad (5.3.2)$$

As mentioned in the introduction, for this case the CFT analysis showed some signs of non-associativity.

In this section we will see that both of these in principle possible non-associative deformations do not lead to any physical effect in on-shell DFT, though the mechanisms turn out to be different for the two cases.

5.3.1 A tri-product for $\check{\mathcal{F}}^{ABC}$

In analogy to the non-associative product for the open string, we consider the DFT tri-product

$$f \triangle g \triangle h = f g h + \frac{\ell_s^4}{6} \check{\mathcal{F}}^{ABC} \mathcal{D}_A f \mathcal{D}_B g \mathcal{D}_C h + O(\ell_s^8). \quad (5.3.3)$$

We assume that f, g, h are scalars under generalized diffeomorphisms and invariant under doubled local Lorentz transformations.

Invoking the strong or closure constraint, $\check{\mathcal{F}}^{ABC}$ and $\mathcal{D}_A f$ transform as scalars under generalized diffeomorphisms so that the tri-product is invariant under the latter. The

anomalous transformation behavior of the tri-product under doubled local Lorentz transformations is

$$\Delta_\Lambda \left(\check{\mathcal{F}}^{ABC} \mathcal{D}_A f \mathcal{D}_B g \mathcal{D}_C h \right) = 3S^{[AD} \mathcal{D}^B \Lambda^{C]}_D \mathcal{D}_A f \mathcal{D}_B g \mathcal{D}_C h \quad (5.3.4)$$

which vanishes directly for the strong constraint and due to (5.2.11) for the closure constraint.

Now consider the effect of the order ℓ_s^4 term under the integral. Performing an integration by parts and using that for both constraints we have $[\mathcal{D}_A, \mathcal{D}_B] = \mathcal{F}^C_{AB} \mathcal{D}_C$, we find

$$\begin{aligned} \int dX e^{-2d} \check{\mathcal{F}}^{ABC} \mathcal{D}_A f \mathcal{D}_B g \mathcal{D}_C h &= \int dX \partial_M (e^{-2d} V^M) + \\ &\int dX e^{-2d} \left[(\mathcal{F}_C - \mathcal{D}_C) \check{\mathcal{F}}^{C[AB]} + \check{\mathcal{F}}^{CD[A} \mathcal{F}_{CD}{}^{B]} \right] f \mathcal{D}_A g \mathcal{D}_B h, \end{aligned} \quad (5.3.5)$$

with

$$V^M = E_A{}^M \check{\mathcal{F}}^{ABC} f \mathcal{D}_B g \mathcal{D}_C h \quad (5.3.6)$$

transforming as a vector under generalized diffeomorphisms. Thus, invoking Stokes theorem this gives a boundary term, which vanishes on well defined compact doubled geometries patched by generalized diffeomorphisms and double Lorentz transformations. Here we have also used the relation

$$\partial_M (E_A{}^M e^{-2d}) = -e^{-2d} \mathcal{F}_A. \quad (5.3.7)$$

The second term can be written as

$$\int dX e^{-2d} \left[\mathcal{G}^{[AB]} - 2S^{M[A} \mathcal{D}^{B]} \mathcal{F}_M \right] f \mathcal{D}_A g \mathcal{D}_B h = 0 \quad (5.3.8)$$

where, due to (5.2.17), $\mathcal{G}^{[AB]}$ vanishes on-shell and the second term vanishes for both the strong and, due to (5.2.8), also for the closure constraint. Thus, we conclude that the order ℓ_s^4 term in the tri-product is a surface term on-shell. In this respect this tri-product is very similar to the open string story.

Matter corrections

However, these equations of motion receive stringy higher derivative corrections, so that the tri-product, i.e. the coefficient $\check{\mathcal{F}}^{ABC}$, needs to be adjusted accordingly. Moreover, coupling DFT to extra matter sources, which, in particular, means *any* additional field

contributing to the energy-momentum tensor, the equations of motion change to

$$2S^{C[A}\mathcal{D}^{B]}\mathcal{F}_C + (\mathcal{F}_C - \mathcal{D}_C)\check{\mathcal{F}}^{C[AB]} + \check{\mathcal{F}}^{CD[A}\mathcal{F}_{CD}{}^{B]} = \mathcal{T}^{AB}. \quad (5.3.9)$$

For instance, including the R-R sector [173, 174], one can put all R-R fields in the spinor representation of $O(D, D)$

$$\mathcal{G} = \sum_n \frac{e^\phi}{n!} G_{i_1 \dots i_n}^{(n)} e_{a_1}{}^{i_1} \dots e_{a_n}{}^{i_n} \Gamma^{a_1 \dots a_n} |0\rangle, \quad (5.3.10)$$

where $\Gamma^{a_1 \dots a_n}$ defines the totally anti-symmetrized product of n Γ -matrices. Then, the R-R contribution to the DFT equation of motion is

$$\mathcal{T}^{AB} = \frac{1}{4} \bar{\mathcal{G}} \Gamma^{AB} \mathcal{G}. \quad (5.3.11)$$

In order to still keep the total derivative property, the only thing one can do is to adjust the tri-product (5.3.3) as

$$f \triangle g \triangle h = \dots + \frac{\ell_s^4}{18} \mathcal{T}^{AB} \left(f \mathcal{D}_{Ag} \mathcal{D}_B h + \mathcal{D}_A f \mathcal{D}_B g h + \mathcal{D}_B f g \mathcal{D}_A h \right). \quad (5.3.12)$$

This means that one already has to introduce a non-trivial two-product as

$$f \triangle_2 g = f \cdot g + \frac{\ell_s^4}{18} \mathcal{T}^{AB} \mathcal{D}_A f \mathcal{D}_B g + O(\ell_s^8). \quad (5.3.13)$$

Let us discuss its effect for the case that one imposes the strong constraint. Below the integral the order ℓ_s^4 correction to this two-product can be written as

$$\begin{aligned} \int dX e^{-2d} \mathcal{T}^{AB} \mathcal{D}_A f \mathcal{D}_B g &= \int dX \partial_M (\dots)^M + \\ &\int dX e^{-2d} \left[(\mathcal{F}_A - \mathcal{D}_A) \mathcal{T}^{AB} - \frac{1}{2} \mathcal{T}^{CD} \mathcal{F}_{CD}{}^B \right] f \mathcal{D}_B g. \end{aligned} \quad (5.3.14)$$

Employing the Bianchi identities (5.2.12) and (5.2.13) as well as the strong or the closure constraint, from (5.3.9) we derive the continuity equation for the energy-momentum tensor

$$(\mathcal{D}_A - \mathcal{F}_A) \mathcal{T}^{AB} + \frac{1}{2} \mathcal{F}_{CD}{}^B \mathcal{T}^{CD} = S^{CA} \mathcal{D}^B \left(\mathcal{D}_A \mathcal{F}_C - \frac{1}{2} \mathcal{F}_A \mathcal{F}_C \right). \quad (5.3.15)$$

Thus, due to the strong constraint the second line in (5.3.14) vanishes and the order ℓ_s^4 correction to the two-product gives a total derivative below the integral. Note that such a two-product implies a two-bracket

$$[x^i, x^j] = \frac{\ell_s^4}{9} \mathcal{T}^{ij}. \quad (5.3.16)$$

We conclude that, due to higher order and source term corrections to the equations of motion, the tri-product needs to be adjusted accordingly. For the matter source term, we showed explicitly that at order ℓ_s^4 this is indeed possible. We find it compelling that the definition of a tri-product and the DFT/string equations of motion are related in this intricate manner. Deforming the underlying geometry in this non-associative way does not effect the on-shell DFT.

5.3.2 A tri-product for \mathcal{F}_{ABC}

Now consider the DFT generalization of the tri-product (5.0.5)

$$f \triangle g \triangle h = f g h + \frac{\ell_s^4}{6} \mathcal{F}_{ABC} \mathcal{D}^A f \mathcal{D}^B g \mathcal{D}^C h + O(\ell_s^8). \quad (5.3.17)$$

Note that once the strong or closure constraint is imposed, the order ℓ_s^4 term in (5.3.17) transforms as a scalar under generalized diffeomorphisms if f, g, h are scalars. In addition this tri-product is also invariant under local double Lorentz transformations. However, a second look reveals that this is trivial as imposing either constraint, one immediately realizes that due to (5.2.6) the whole order ℓ_s^4 term actually *vanishes*. Thus, in this constrained DFT framework this tri-product is actually trivial.

For illustrative purposes, nevertheless let us apply a partial integration to the tri-product (5.3.17) written below an integral. The order ℓ_s^4 term can be written as

$$\begin{aligned} \int dX e^{-2d} \mathcal{F}_{ABC} \mathcal{D}^A f \mathcal{D}^B g \mathcal{D}^C h &= \int dX \partial^M (\dots)_M - \\ &\int dX e^{-2d} \left[(\mathcal{D}^C - \mathcal{F}^C) \mathcal{F}_{CAB} \right] \mathcal{D}^A f \mathcal{D}^B g h \end{aligned} \quad (5.3.18)$$

where the term in the last line can be written as

$$\int dX e^{-2d} \left[\mathcal{Z}_{AB} - 2\mathcal{D}_{[A} \mathcal{F}_{B]} \right] \mathcal{D}^A f \mathcal{D}^B g h. \quad (5.3.19)$$

Here we have used $\mathcal{F}_{MN[A} \mathcal{F}^{MN}_{B]} = 0$. Consistently, due to the Bianchi-identity (5.2.13) and the relation (5.2.8) this expression vanishes for both constraints. Since the terms appearing in this computation are related to the ones appearing in a topological Bianchi identity and not a dynamical equation of motion, one might expect that there are no stringy higher order derivative corrections to the non-constant tri-product parameter \mathcal{F}_{ABC} .

Comments on relaxing the closure constraint

Relaxing even the closure constraint is the only option to get a non-trivial tri-product (5.3.17). For compact configurations it is clear that string theory contains momentum

and winding modes not subject to the weak and consequently the strong constraint. For instance, for a toroidal compactification, the level matching condition becomes

$$L_0 - \bar{L}_0 = \alpha' p \cdot w + N - \bar{N} = 0 \quad (5.3.20)$$

where N and \bar{N} denote the number of left- and right-moving oscillator excitations. Including these modes is expected to go beyond the realm of DFT.

Another way of relaxing the closure constraint could be by splitting the fluxes into backgrounds and fluctuations as well as relaxing the strong and closure constraint between the two. Whether this is an allowed relaxation in DFT remains to be seen and is beyond the scope of this thesis. Here we just discuss its consequences for the tri-product.

Independent of how the constraints are relaxed actually, let us now discuss the consequences for the tri-product. Up to boundary terms, after partially integrating the order ℓ_s^4 term under the integral we get

$$\int dX e^{-2d} \left[(\mathcal{D}^C - \mathcal{F}^C) \mathcal{F}_{C[AB]} + 2 \Omega_{CD[A} \mathcal{F}_{B]}{}^{CD} \right] (\mathcal{D}^A f) (\mathcal{D}^B g) h. \quad (5.3.21)$$

The additional term compared to (5.3.18) arises from the Ω term in the commutator (5.2.16) when violating closure. Taking into account that, in string theory non-associativity should still be vanishing at least on shell, we can imagine two ways to proceed from here.

First, we can require a new constraint

$$\zeta_{AB} \mathcal{D}^A f \mathcal{D}^B g = 0 \quad (5.3.22)$$

with

$$\zeta_{AB} = (\mathcal{D}^C - \mathcal{F}^C) \mathcal{F}_{C[AB]} + 2 \Omega_{CD[A} \mathcal{F}_{B]}{}^{CD} \quad (5.3.23)$$

that is weaker than the closure constraint. The second possibility is to cancel these terms by an appropriately adjusted tri-product

$$\begin{aligned} f \triangle g \triangle h &= f g h + \frac{\ell_s^4}{6} \mathcal{F}_{ABC} \mathcal{D}^A f \mathcal{D}^B g \mathcal{D}^C h \\ &\quad + \frac{\ell_s^4}{18} \zeta_{AB} \left(f \mathcal{D}^A g \mathcal{D}^B h + \mathcal{D}^A f \mathcal{D}^B g h + \mathcal{D}^B f g \mathcal{D}^A h \right). \end{aligned} \quad (5.3.24)$$

Note that one can rewrite the adjusted tri-product (5.3.24) as

$$f \triangle g \triangle h = f g h + e^{2d} \partial_M \left(\frac{\ell_s^4}{6} E_A{}^M e^{-2d} \mathcal{F}_{ABC} f \mathcal{D}^B g \mathcal{D}^C h + \text{cycl}_{f,g,h} \right) \quad (5.3.25)$$

showing that it is really designed to give a boundary term below the integral. One can

show that also the induced two-product gives a boundary term if written under an integral.

Summarizing, to relax the closure constraint one can either impose (5.3.22) or define the tri-product deformation trivially as a total derivative. In both cases one formally has non-vanishing brackets (5.0.2) and (5.0.3) that leave no trace under an action integral.

Holonomic basis

In order to see more concretely what is happening here, let us consider as an example a holonomic basis with $B_{ab} = 0$, $f_{ab}^c = 0$ and $\tilde{f}^{ab}_c = 0$. In this case one finds

$$\begin{aligned}
 \mathcal{F}_{ABC} \mathcal{D}^A f \mathcal{D}^B g \mathcal{D}^C h &= R^{ijk} \partial_i f \partial_j g \partial_k h + \\
 &\quad Q_k^{ij} \left(\partial_i f \partial_j g (\tilde{\partial}^k + \beta^{kl} \partial_l) h + \text{cycl}_{f,g,h} \right) \\
 &= 3 \left(\tilde{\partial}^{[i} \beta^{jk]} + \beta^{[im} \partial_m \beta^{jk]} \right) \partial_i f \partial_j g \partial_k h \\
 &\quad - 3 \left(\beta^{[im} \partial_m \beta^{jk]} \right) \partial_i f \partial_j g \partial_k h + \partial_k \beta^{ij} \left(\partial_i f \partial_j g \tilde{\partial}^k h + \text{cycl}_{f,g,h} \right)
 \end{aligned} \tag{5.3.26}$$

where we have split the R -flux as

$$R^{ijk} = \hat{R}^{ijk} + R_{\text{cl}}^{ijk} = 3 \left(\tilde{\partial}^{[i} \beta^{jk]} + \beta^{[im} \partial_m \beta^{jk]} \right). \tag{5.3.27}$$

Therefore, the second and third term cancel and the sum of the first and fourth vanish by the constraint. In particular, this means that in DFT the classical part $R_{\text{cl}}^{ijk} = \beta^{[im} \partial_m \beta^{jk]}$ does not contribute to the tri-product.

In order to derive the tri-bracket among three coordinates, let us choose for the three functions $f = x^i$, $g = x^j$ and $h = x^k$. Without imposing neither the strong nor the closure constraint³ the resulting tri-bracket is then given by

$$[x^i, x^j, x^k] = \ell_s^4 \hat{R}^{ijk}, \tag{5.3.28}$$

and in particular only contains the R -flux \hat{R}^{ijk} .

Let us also consider the general commutator (5.0.3) for the case that both Q - and R -flux is present in more detail. Our DFT analysis suggests that the commutator for general functions should be defined as

$$\begin{aligned}
 -\frac{3i\hbar}{\ell_s^4} [f, g] &= R^{ijk} \partial_i f \partial_j g \partial_k + Q_k^{ij} \left(\partial_i f \partial_j g (\tilde{\partial}^k + \beta^{kl} \partial_l) + \right. \\
 &\quad \left. (\tilde{\partial}^k + \beta^{kl} \partial_l) f \partial_i g \partial_j + \partial_j f (\tilde{\partial}^k + \beta^{kl} \partial_l) g \partial_i \right).
 \end{aligned} \tag{5.3.29}$$

³The CFT computations performed in [112, 113, 150–152] were not imposing any constraints so that they can be considered to be reliable for the compact torus case for which the level matching condition is (5.3.20).

Inserting the definition of the R -flux (5.3.27), again the term R_{cl}^{ijk} completely cancels against terms appearing in the Q -flux contribution and we are left with

$$-\frac{3i\hbar}{\ell_s^4} [f, g] = \hat{R}^{ijk} \partial_i f \partial_j g \partial_k + Q_k^{ij} \left(\partial_i f \partial_j g \tilde{\partial}^k + \tilde{\partial}^k f \partial_i g \partial_j + \partial_j f \tilde{\partial}^k g \partial_i \right). \quad (5.3.30)$$

Note that, invoking the constraint, the commutator vanishes. Computing the commutation relations for the coordinate functions, without imposing any constraint, one finds

$$\begin{aligned} [x^i, x^j] &= i \frac{\ell_s^4}{3\hbar} \left(\hat{R}^{ijk} \partial_k + Q_k^{ij} \tilde{\partial}^k \right), \\ [x^i, \tilde{x}_k] &= -i \frac{\ell_s^4}{3\hbar} Q_k^{ij} \partial_j. \end{aligned} \quad (5.3.31)$$

Thus, DFT suggests that the interpretation of the commutation relation (5.0.3) in terms of derivatives is (5.3.31). In particular, the contribution R_{cl}^{ijk} drops out and all commutators vanish after imposing any constraint.

Higher order corrections

At leading order in derivatives of \mathcal{F}_{ABC} there is a natural candidate for all the order in ℓ_s^4 tri-product, namely

$$(f \triangle g \triangle h)(X) = \exp \left(\frac{\ell_s^4}{6} \mathcal{F}_{ABC} \mathcal{D}_{X_1}^A \mathcal{D}_{X_2}^B \mathcal{D}_{X_3}^C \right) f(X_1) g(X_2) h(X_3) \Big|_X. \quad (5.3.32)$$

At leading order in $(\mathcal{D}\mathcal{F}_{ABC})$, except fgh , all terms give a total derivative below the integral. The appearing derivatives can be canceled by defining the overall tri-product as

$$\begin{aligned} f \blacktriangle g \blacktriangle h &= f \triangle g \triangle h + \sum_{k=2}^{\infty} \frac{\ell_s^{4k}}{3 \cdot 6^k k!} \left\{ \mathcal{F}_{A_1 B_1 D} \mathcal{D}^D (\mathcal{F}_{A_2 B_2 C_2} \dots \mathcal{F}_{A_k B_k C_k}) \right. \\ &\quad \left. \left((\mathcal{D}^{A_1} \dots \mathcal{D}^{A_k} f)(\mathcal{D}^{B_1} \dots \mathcal{D}^{B_k} g)(\mathcal{D}^{C_2} \dots \mathcal{D}^{C_k} h) + \text{cycl}_{\{f, g, h\}} \right) \right\}. \end{aligned} \quad (5.3.33)$$

This product is designed to satisfy

$$\int dX e^{-2d} f \blacktriangle g \blacktriangle h = \int dX e^{-2d} f g h. \quad (5.3.34)$$

A possible generalization of the tri-product to the product of K functions is presented in the appendix A.

5.3.3 Non-associativity in heterotic DFT

Recall that heterotic DFT can be considered as an extension of DFT with gauge fields manifested and global symmetry group $O(D, D + n)$. We devote this section to the non-associativity in heterotic DFT framework. Note that the tri-bracket $[x^i, x^j, x^k]$ for the coordinates is governed by the non-geometric flux coupled to just ordinary derivatives. Thus, we are focusing on the term

$$\mathcal{F}_{ABC} D^A f D^B g D^C h = \rho^{ijk} \partial_i f \partial_j g \partial_k h + \dots \quad (5.3.35)$$

which for usual DFT was just $\rho_{bos}^{ijk} = 3\tilde{\partial}^{[i}\beta^{jk]}$. The natural expectation is that, in the heterotic case, this gets generalized to the gauge invariant combination⁴

$$\rho_H^{ijk} = 3\left(\tilde{\partial}^{[i}\beta^{jk]} - \tilde{\partial}^{[i}\tilde{A}^{j\gamma}\tilde{A}^{k]}_{\gamma}\right). \quad (5.3.36)$$

However, evaluating (5.3.35) for a holonomic non-geometric frame, one finds

$$\rho^{ijk} = 3(\tilde{\partial}^{[i}E^{j]}_A E^{Ak]} = -3\left(\tilde{\partial}^{[i}\beta^{jk]} + \tilde{\partial}^{[i}\tilde{A}^{j\gamma}\tilde{A}^{k]}_{\gamma}\right), \quad (5.3.37)$$

showing that the relative sign between the two terms on the right hand side of (5.3.37) is different. As a consequence, this object ρ^{ijk} is not invariant under \tilde{A} gauge transformations $\tilde{A}^i_{\alpha} = \tilde{A}^i_{\alpha} + \tilde{\partial}^i \lambda_{\alpha}$, unless the non-geometric gauge flux $\tilde{G}_{\alpha}^{ij} = \tilde{\partial}^{[i}\tilde{A}^{j]}_{\alpha}$ vanishes. We observe that this sign flip can be reconciled with heterotic DFT by defining instead

$$f \triangle g \triangle h = f g h + \mathcal{F}_{ABC}^H D^A f D^B g D^C h + \dots \quad (5.3.38)$$

with

$$\mathcal{F}_{ABC}^H(\beta, \tilde{A}) = \mathcal{F}_{ABC}(\beta, \tilde{A}) - 2\mathcal{F}_{ABC}(\beta, \tilde{A} = 0). \quad (5.3.39)$$

5.4 Summary and discussion

In this chapter, using the flux formulation of DFT, we have analyzed the consequences of introducing non-associativity via a non-trivial tri-product for the functions on the manifold. We analyzed two different such non-associative deformations. For the first one the deforming flux was given by $\check{\mathcal{F}}^{ABC}$ and for the second one by \mathcal{F}_{ABC} . The first case is the DFT generalization of the H^{ijk} -flux deformation and the second one the generalization of the R^{ijk} -flux deformation.

We argued from a CFT point of view that non-associative deformations should not lead to any physical effect on shell. Note that in the open string case, the situation is different.

⁴We confirmed this behavior by performing a CFT analysis along the lines of [113].

There the DBI action can be expressed in the Seiberg-Witten limit as a non-commutative gauge theory and the higher orders in the star-product really contribute physical terms to the deformed action. However, also here cyclicity and associativity are preserved on-shell.

The $\check{\mathcal{F}}^{ABC}$ flux case is conceptually very close to its open string analogue. We found that, at leading order in ℓ_s^4 , the deformation gives a boundary term under the integral if the DFT equations of motion are satisfied and the strong or closure constraint is employed. We showed that for additional matter contributions the tri-product can be adjusted accordingly. This led to a new deformation of the two-product, whose on-shell triviality was guaranteed by the continuity equation of the energy momentum tensor. This means that on-shell DFT or string theory cannot distinguish between an ordinary smooth geometry and a fuzzy one with fundamental tri-bracket

$$[x^i, x^j, x^k] = \ell_s^4 H^{ijk}. \quad (5.4.1)$$

Even though from [112, 113, 150–152] we do not have any evidence for such a non-associative behavior of the coordinates, we find this a remarkable property of DFT. Turning the logic around, up to the dilaton sector, one can derive the DFT equations of motion from the concept of the *absence of on-shell non-associativity*. We emphasize, that in the flux formulation of DFT also the gravity part is fully encoded in the generalized three-form flux. This is very similar to the familiar magnetic monopole example discussed in the first section.

The \mathcal{F}_{ABC} flux case is the one where non-associativity was expected. We realized that in the DFT framework this tri-product actually vanishes after imposing either the strong or the closure constraint⁵. Therefore, in order to get something non-trivial even the closure constraint need to be weakened. Only then one could obtain a non-associative deformation of the target space action with the three-bracket for the internal coordinates x^i being

$$[x^i, x^j, x^k] = \ell_s^4 \hat{R}^{ijk}. \quad (5.4.2)$$

Again note that the \hat{R}^{ijk} only contain the winding part of the full R -flux, the classical part has canceled out.

On a more speculative level, we also proposed a generalization of the tri-product to higher orders in ℓ_s^4 and for products of K -terms.

Summarizing, the resolution to the initially raised paradox is that one can have a non-associative deformation of the target space, while nothing of it is immediately apparent in the effective string and DFT actions for the massless modes. Deforming the product to a tri-product we have found two different ways how such a deformation can become trivial on-shell.

One could imagine that, due to the finite size and resolution of the string, there exists a certain non-associative deformation of the target space that is “under the radar” of the

⁵Along this line, we also presented the non-associativity in heterotic DFT framework.

string. Therefore, string theory can very well admit such non-geometric spaces as honest backgrounds. We illustrate this stringy equivalence in Figure 5.1.

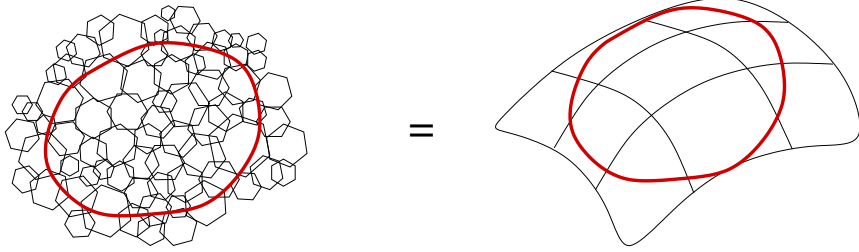


Figure 5.1: Stringy equivalence between fuzzy non-associative geometry and smooth Riemannian geometry.

It would be interesting to carry out a similar analysis for the non-commutative closed string star product defined on phase space, which was introduced and discussed in [154,155]. Moreover, one could study what other deeper conceptual consequences the existence of such a non-geometric regime of string theory might have. For future investigations, we found it interesting to study whether this can be generalized to string field theory with massive string states included, and analogous structure for M-theory.

String Phenomenology with Non-Geometries

In this chapter, we study the string phenomenology application of non-geometric backgrounds. As introduced in chapter 3, we will focus on type IIB orientifold compactifications with (non)-geometric fluxes turned on. The non-geometric fluxes arise as the T-dual and/or S-dual of the geometric fluxes.

Motivated by realizing single field F-term axion monodromy inflation [175–177], a scheme of high scale supersymmetry breaking was proposed in [134]. The inflaton therein was an axion obtain a polynomial potential from a tree-level background fluxes. This led to large field inflation models with proper tensor-to-scalar ratio, an inflationary scale in the order of the GUT scale and an inflaton mass of order 10^{13} GeV. Since for single field inflation, the inflaton has to be the lightest scalar field while all other moduli should better acquire their masses already at tree-level. For type IIB orientifold compactifications on Calabi-Yau three-folds (CY_3) this means in particular that all closed string moduli, namely the axio-dilaton as well as the complex structure and Kähler moduli, should be stabilized by geometric and non-geometric fluxes. Closed string moduli stabilization with solely fluxes was discussed in [134] while its application to axion inflation was further studied in [178].

Recall that one of the main results of [134] is, by turning on $(n+1)$ fluxes for n moduli, the resulting F-term scalar potential admits so-called scaling type non-supersymmetric AdS minima with the desired properties. Here scaling type means that the values of the moduli in the minimum as well as all the mass scales are determined by ratios of products of fluxes. This allows for parametric control of these quantities. This is important in order to argue for the self-consistency of the moduli stabilization scheme, i.e. that eventually the moduli are stabilized in their perturbative regime and that, e.g. the moduli masses are separated from the string and Kaluza-Klein scales.

Conceptually, the induced F-term scalar potential is related to the one of $N = 2$ gauged supergravity by an orientifold projection breaking $N = 2$ down to $N = 1$ [139]. Recently, it was explicitly shown in [135] that the same potential also arises by appropriate dimensional reduction of DFT on a CY_3 equipped with fluxes. Moreover, in DFT framework, a D-term potential emerges when there are abelian gauge fields present, coming from the dimensional reduction of the R-R four-form on an orientifold even three-cycle of the CYs [114].

We note that throughout the work in [134], the flux-scaling AdS vacua demand for uplift to Minkowski or to de Sitter vacua, for instance by introducing an $\overline{\text{D3}}$ -brane as in the KKLT scenario [20]. As a fairly new and significant development, it has been recently pointed out that this often employed $\overline{\text{D3}}$ -brane uplift mechanism can be described within supergravity by a nilpotent superfield [22, 179, 180] and the vacua are argued to be metastable [181]. However, a concrete example in [134], showed that a naive uplift of flux-scaling AdS vacua by introducing an $\overline{\text{D3}}$ -brane in a warped throat does not work. Indeed, by increasing the warp factor, the minimum got destabilized before the cosmological constant vanished. However, for string theory to provide a reliable description of inflation, one has to explain the cosmological constant in a self-consistent string compactification.

Since the question of uplifting is clearly a very important unsettled issue in the flux-scaling scenario, it is the purpose of this chapter to investigate this problem more closely. First, for the $\overline{\text{D3}}$ -brane case we will find that adding the tension of this brane to the flux induced F-term potential can lead to new flux-scaling solutions that are of Minkowski/de Sitter type. Second, for $h_+^{2,1} > 0$ there is an additional positive semi-definite D-term contribution to the scalar potential [114, 135] that in principle could help with increasing the cosmological constant at the minimum. We will show both of the two approaches give rise to de Sitter vacua. However, it is worthwhile to emphasize that these are not continuous uplifts of initial AdS minima from moduli stabilization, but new minima lying on a different branch in the landscape.

As mentioned above, the motivation for moduli stabilization in the flux-scaling scenario was the stringy realization of axion monodromy inflation. Therefore, we will revisit the possibility of realizing axion monodromy inflation based on the de Sitter vacua we obtain. We will show that for integer quantized fluxes, it is persistently difficult to obtain all mass scales in the right order, namely

$$M_s > M_{\text{KK}} > M_{\text{inf}} > M_{\text{mod}} > H_{\text{inf}} > M_\theta, \quad (6.0.1)$$

where θ denotes the inflaton. However, it is known that the perturbative corrections to the prepotential of the complex structure moduli lead to a redefinition of the fluxes so that some of them are able to obtain rational shifts for their values. Phenomenologically scanning over such rational values, we present models in which the above hierarchy is indeed fulfilled.

This chapter is organized as follows: In section 6.1 we present examples of uplifted flux-scaling vacua. We discuss one model with an $\overline{\text{D3}}$ -brane uplift and another one with a D-term uplift. We also show that by changing the warp factor for the former example, one can interpolate between AdS and dS vacua. In section 6.2 we analyze the realization of axion monodromy inflation in the model with D-term uplift.

6.1 Uplifting to de Sitter

In this section we investigate whether, by adding additional positive definite contributions to the F-term scalar potential, one can directly find scaling type, non-supersymmetric metastable minima that are of de Sitter or Minkowski type.

Before turning to the uplift analysis in the next sections we state our conventions and notation for the different mass scales. For the Planck mass we take $M_{\text{Pl}} \sim 2.435 \cdot 10^{18}$ GeV, and for the string mass scale $M_s = (\alpha')^{1/2}$. In terms of M_{Pl} , the string and Kaluza-Klein scales are given by

$$M_s = \frac{\sqrt{\pi} M_{\text{Pl}}}{s^{\frac{1}{4}} \mathcal{V}^{\frac{1}{2}}}, \quad M_{\text{KK}} = \frac{M_{\text{Pl}}}{\sqrt{4\pi} \mathcal{V}^{\frac{1}{4}}}, \quad (6.1.1)$$

where $s = e^{-\phi}$ and \mathcal{V} is the volume of the Calabi-Yau manifold in Einstein frame in string units. The moduli masses are determined by the eigenvalues of the canonically normalized mass matrix, which is defined as

$$(M^2)_j^i = K^{ik} V_{kj}, \quad (6.1.2)$$

where $V_{ij} = \frac{1}{2} \partial_i \partial_j V$. Finally, the gravitino mass reads

$$M_{3/2}^2 = e^{K_0} |W_0|^2 \frac{M_{\text{Pl}}^2}{4\pi} \quad (6.1.3)$$

where K_0 and W_0 stand for the Kähler and superpotential evaluated at the minima.

Recall that in the KKLT [20] or LARGE volume scenario [18, 182], one starts with an AdS minimum and adds the contributions of an $\overline{\text{D3}}$ -brane in a warped throat. By varying the coefficient of this contribution, i.e. the warp factor, one can continuously shift the cosmological constant in the minimum from the negative AdS value to positive dS values. In the first part of this section we analyze, in a concrete example, the effect of adding an $\overline{\text{D3}}$ -brane to the F-term flux-induced potential.

In (3.2.35) we have recalled that for $h_+^{2,1} > 0$ the scalar potential receives an additional positive definite D-term contribution (3.2.36). In the second part of this section, we will try to uplift AdS vacua by turning on the fluxes contributing to this D-term and study the possibility of realizing de Sitter vacua and thus possible inflation models.

6.1.1 Uplift via $\overline{\text{D3}}$ -brane

A common mechanism to uplift AdS vacua preserving stability is to introduce an $\overline{\text{D3}}$ -brane at a warped throat as introduced in [20, 183]. This generates a contribution to the scalar

potential of the form

$$V_{\text{up}} = \frac{A}{\mathcal{V}^{\frac{4}{3}}} \frac{M_{\text{Pl}}^4}{4\pi}, \quad (6.1.4)$$

where A a positive constant depending on the warp factor in the throat. Following this procedure, we now consider a concrete example to show the $\overline{\text{D3}}$ -brane contribution to the scalar potential for a scaling type minimum.

A stable AdS minimum

Consider a CY manifold with $h_+^{1,1} = 1$, $h_-^{1,1} = 0$, $h_-^{2,1} = 1$ and $h_+^{2,1} = 0$, the total scalar potential after tadpole cancellation is given just by the F-term. The tree-level Kähler potential reads

$$K = -\log(S + \bar{S}) - 3\log(T + \bar{T}) - 3\log(U + \bar{U}), \quad (6.1.5)$$

and the superpotential is given by

$$W = -ifU + ih_0S - 3ihSU^2 - iqT. \quad (6.1.6)$$

According to (3.2.28), $\mathfrak{f}_1 = f$, $\tilde{h}^1 = -h$ and $q_0^1 = q$. For the following discussion, we denote $S = s + ic$, $T = \tau + i\rho$ and $U = v + iu$.

At first in the absence of the $\overline{\text{D3}}$ -brane, we find a completely stable supersymmetric AdS vacuum of scaling type. The axionic moduli are fixed at $\rho = c = u = 0$, whereas the saxions are fixed at

$$s = -\frac{5^{1/2}}{4} \frac{f}{(hh_0)^{1/2}}, \quad v = \frac{5^{1/2}}{3} \left(\frac{h_0}{h}\right)^{1/2}, \quad \tau = -\frac{5^{1/2}f}{2q} \left(\frac{h_0}{h}\right)^{1/2}. \quad (6.1.7)$$

To be in the physical regime, we choose fluxes

$$f < 0, \quad h_0 > 0, \quad h > 0, \quad q > 0. \quad (6.1.8)$$

To stay consistently in the perturbative regime, one can choose $|f| \gg 1$ whereas all other fluxes of order $\mathcal{O}(1)$. The value of the scalar potential at the minimum is determined analytically to be

$$V_0 = -\frac{9}{5^{5/2}} \frac{q^3 h^{5/2}}{4 f^2 h_0^{3/2}} \frac{M_{\text{Pl}}^4}{4\pi}. \quad (6.1.9)$$

The normalized moduli masses are

$$M_{\text{mod}}^2 = \mu_i \frac{q^3 h^{5/2}}{f^2 h_0^{3/2}} \frac{M_{\text{Pl}}^2}{4\pi}, \quad (6.1.10)$$

with coefficients

$$\mu_i = \{0.4039, 0.2414, 0.1208; 0.5699, 0.1341, 0.0442\}. \quad (6.1.11)$$

The first three entries are saxionic while the last three are axionic. We see that here the lightest state is axionic.

Uplift to a Minkowski minimum

Now, we implement the uplift term from an $\overline{\text{D3}}$ -brane in the throat, namely (6.1.4). We find a stable Minkowski minimum with the axions kept at the origin, while the saxions are shifted to

$$s = \frac{1}{3^{3/4}} \frac{f}{(h h_0)^{1/2}}, \quad v = \frac{1}{3^{1/4}} \left(\frac{h_0}{h} \right)^{1/2}, \quad \tau = \frac{f}{3^{1/4} q} \left(\frac{h_0}{h} \right)^{1/2}. \quad (6.1.12)$$

The warp dependent parameter A is determined to be

$$A = \frac{3^{1/4}}{2} \frac{q h^{3/2}}{h_0^{1/2}}. \quad (6.1.13)$$

In order to have positive saxion vacuum expectation values at the minimum, the fluxes have to be in the regime

$$f > 0, \quad h_0 > 0, \quad h > 0, \quad q > 0. \quad (6.1.14)$$

Furthermore, $A > 0$ is required. Note that the sign of f is different from the supersymmetric AdS minimum, it is clear that this Minkowski vacuum is not literally a continuous uplift of the former, but constitutes a new non-supersymmetric, scaling type Minkowski vacuum. The normalized moduli masses have the same flux dependence as in (6.1.10) for the AdS vacuum, whereas the numerical coefficients shift to

$$\mu_i = \{0.8034, 0.4868, 0.03942; 1.5559, 0.2116, 0.0811\}. \quad (6.1.15)$$

Moreover, the lightest state is a linear combination of saxions instead.

Employing the expressions given at the end of section 3.2.3, we compute the other relevant mass scales. The gravitino mass has the same scaling behavior as in (6.1.10) with

coefficient $\mu_{3/2} = 0.3135$. Moreover, the Kaluza-Klein and string scales are given by

$$M_s^2 = \frac{3^{3/4}\pi}{2^{3/2}} \frac{q^{3/2}h}{f^2 h_0^{1/2}} M_{\text{Pl}}^2, \quad M_{\text{KK}}^2 = \frac{3^{1/2}}{16\pi} \frac{q^2 h}{f^2 h_0} M_{\text{Pl}}^2 \quad (6.1.16)$$

so that the relevant ratios are determined to be

$$\frac{M_{\text{KK}}^2}{M_s^2} = \frac{1}{2^{5/2} 3^{1/4} \pi^2} \left(\frac{q}{h_0} \right)^{1/2}, \quad \frac{M_{\text{mod},i}^2}{M_{\text{KK}}^2} = \frac{2^2 \mu_i}{3^{1/2}} \frac{q h^{3/2}}{h_0^{1/2}}. \quad (6.1.17)$$

Requiring $h, q \sim \mathcal{O}(1)$ and $h_0 \sim f \gg 1$ we observe that parametrically the moduli are in their perturbative regime and one can achieve the required mass hierarchy $M_s \gtrsim_p M_{\text{KK}} \gtrsim_p M_{\text{mod}}$. However, for $h_0 \gg 1$ we obtain $A \ll 1$ which indicates for large warping in the Calabi-Yau throat. Another characteristic feature of this model is that the fluxes do not contribute to the D7-brane tadpole whereas

$$N_{\text{D3}} = f h. \quad (6.1.18)$$

Note that, in the supersymmetric AdS vacuum $N_{\text{D3}} < 0$, while in the Minkowski vacuum $N_{\text{D3}} > 0$. Increasing f clearly suggests a larger flux tadpole.

This example shows that adding an $\overline{\text{D3}}$ -brane to the fluxed CY manifold the scalar potential admits new stable scaling type Minkowski vacua. Such vacua could serve as candidate for F-term axion monodromy inflation along the lines proposed in [134, 178, 184].

Uplift to a de Sitter minimum

By choosing the parameter A in the $\overline{\text{D3}}$ -brane potential larger than (6.1.13), one expects to also get a de Sitter vacuum. Let us analyze this in an expansion in $\Lambda = V_0$, namely the value of the scalar potential in the minimum. Indeed changing the value of A , in the minimum, the axions are kept at the origin while the saxions shift to

$$\begin{aligned} s &= \frac{1}{3^{3/4}} \frac{f}{(h h_0)^{1/2}} + \frac{2^4 \cdot 7}{3^{5/2}} \frac{f^3 h_0}{q^3 h^3} \Lambda + \mathcal{O}(\Lambda^2), \\ v &= \frac{1}{3^{1/4}} \left(\frac{h_0}{h} \right)^{1/2} - \frac{2^4}{3^2} \frac{f^2 h_0^2}{q^3 h^3} \Lambda + \mathcal{O}(\Lambda^2), \\ \tau &= \frac{f}{3^{1/4} q} \left(\frac{h_0}{h} \right)^{1/2} + \frac{2^4 \cdot 13}{3^2} \frac{f^3 h_0^2}{q^4 h^3} \Lambda + \mathcal{O}(\Lambda^2). \end{aligned} \quad (6.1.19)$$

The parameter A is determined to be

$$A = \frac{3^{1/4}}{2} \frac{q h^{3/2}}{h_0^{1/2}} + \frac{2^2}{3^{1/2}} \frac{f^2 h_0}{q^2 h} \Lambda + \mathcal{O}(\Lambda^2). \quad (6.1.20)$$

In Figure 6.1 we illustrate the scalar potential leading to a de Sitter minimum. Even though, for simplicity, only the dependence on a single modulus (here τ) is shown, the plot behaves as expected from KKLT. In particular, the dS minimum is only metastable as the potential goes to zero for large τ . We show the scalar potential $V(\tau)$ in units of $\frac{M_{\text{Pl}}^4}{4\pi}$ for $\{s, v\}$ and the axions take their minimum value. The fluxes are $h_0 = 10$, $h = q = 1$, $f = 5$. A is chosen to give a de Sitter minimum.

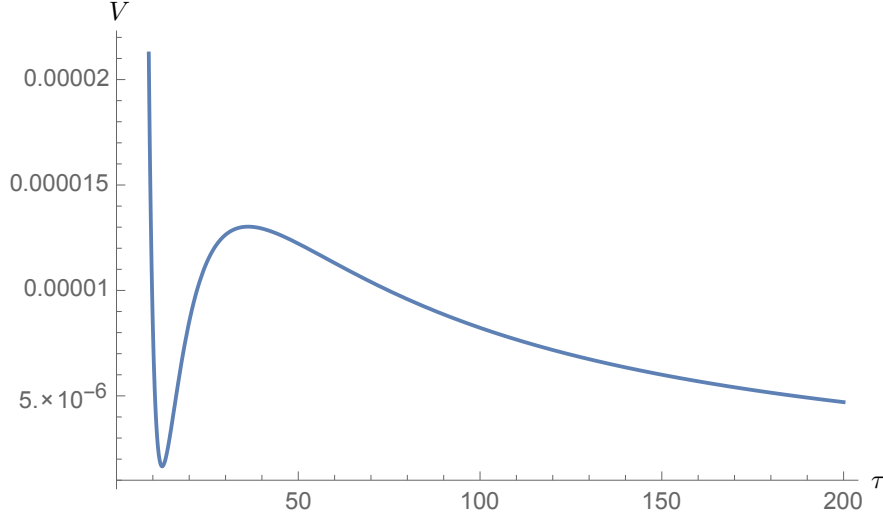


Figure 6.1: The scalar potential $V(\tau)$ leading to a de Sitter minimum.

The upshot is that for small $|\Lambda|$, one can continuously interpolate from an AdS minimum to a de Sitter one. However one needs to be careful that at certain critical values of $|\Lambda|$, the vevs of the saxions in (6.1.19) can become negative and thus unphysical. The normalized moduli masses also get corrected at linear order in Λ ,

$$M_{\text{mod}}^2 = \left(\mu_i \frac{q^3 h^{5/2}}{f^2 h_0^{3/2}} - \tilde{\mu}_i \Lambda + \mathcal{O}(\Lambda^2) \right) \frac{M_{\text{Pl}}^2}{4\pi}, \quad (6.1.21)$$

with coefficients

$$\mu_i = \{0.8034, 0.4868, 0.03942; 1.5559, 0.2116, 0.0811\}, \quad (6.1.22)$$

and

$$\tilde{\mu}_i = \{46.5221, 34.4038, 6.1852; 125.614, 6.5749, 3.6748\}. \quad (6.1.23)$$

Note that the linear contribution of a positive cosmological constant decreases the mass of all the moduli. Thus, for too large Λ , we expect the appearance of tachyonic states. The

Kaluza-Klein and string scale also receive corrections so that the relevant ratios become

$$\begin{aligned}\frac{M_{\text{KK}}^2}{M_s^2} &= \frac{1}{2^{5/2} 3^{1/4} \pi^2} \left(\frac{q}{h_0} \right)^{1/2} - \frac{2^{3/2}}{3\pi^2} \frac{f^2 h_0}{q^{5/2} h^{5/2}} \Lambda + \mathcal{O}(\Lambda^2), \\ \frac{M_{\text{mod},i}^2}{M_{\text{KK}}^2} &= \frac{2^2}{3^{1/2}} \mu_i \frac{q h^{3/2}}{h_0^{1/2}} + \frac{2^2}{3^3} (2^5 \cdot 13 \cdot 3^{3/4} \mu_i + 3^{5/2} \tilde{\mu}_i) \frac{f^2 h_0}{q^2 h} \Lambda + \mathcal{O}(\Lambda^2).\end{aligned}\tag{6.1.24}$$

Finally, we observe that the scaling behavior for all quantities is corrected at subleading order in Λ .

6.1.2 D-term uplift

In this section, we investigate the second possibility for uplift, by taking the naturally appearing D-terms (3.2.36) into account. These positive semi-definite contributions only depend on the saxionic modes and therefore do not change the axion stabilization.

D-term potential from $h_+^{2,1}$ vector multiplets

We will discuss the form of this D-term in more detail. We focus on the case $h_+^{2,1} = 1$, and $h_-^{2,1} = 1$. To simplify we also take $h_+^{1,1} = 1$ and $h_-^{1,1} = 0$. In the notation of section 3.2.3 we turn on the fluxes

$$f_{\hat{1}0} = r, \quad f_{\hat{1}1} = g, \tag{6.1.25}$$

whereas $\tilde{f}_{\hat{1}0} = 0$ and $\tilde{f}_{\hat{1}1} = 0$. The D-term potential is then given by

$$V_D = -\frac{M_{\text{Pl}}^4}{2} \frac{D_{\hat{1}}^2}{\text{Im}\mathcal{N}}, \tag{6.1.26}$$

where $\mathcal{N} = \mathcal{N}_{\hat{1}\hat{1}}$ will be determined shortly, and $D_{\hat{1}}$ reads

$$D_{\hat{1}} = \frac{gt}{\mathcal{V}} - r e^\phi = \frac{3}{\tau} \left(g - \frac{r\tau}{3s} \right). \tag{6.1.27}$$

Here we have used $\mathcal{V} = \frac{1}{6} \kappa t^3$, $T + \bar{T} = \kappa t^2 = 2\tau$, and $s = e^{-\phi}$.

Let us now compute the remaining ingredient $\text{Im}\mathcal{N}$. As explained in [133], when properties of the orientifold projection are taken into account, the relation between the relevant period matrix elements and the prepotential reduces to

$$\mathcal{N}_{\hat{\lambda}\hat{\sigma}} = \bar{F}_{\hat{\lambda}\hat{\sigma}}. \tag{6.1.28}$$

On the right hand side the complex structure deformations associated to $h_+^{2,1}$ are set to

zero. Working in the large complex structure limit, the prepotential in our case can be expressed as

$$F = \frac{1}{X^0} (d_{111}X^3 + 3d_{1\hat{1}\hat{1}}XZ^2) , \quad (6.1.29)$$

where $X = X^1$ and $Z = X^{\hat{1}}$. The form of the cubic prepotential follows imposing that under the orientifold involution X and X^0 are even, whereas Z is odd. The complex structure parameter associated to $h_-^{2,1} = 1$ is defined as

$$U = -i \frac{X}{X^0} = v + iu . \quad (6.1.30)$$

We then find

$$\text{Im} \mathcal{N} = -3d_{1\hat{1}\hat{1}} (U + \bar{U}) = -6d_{1\hat{1}\hat{1}} v . \quad (6.1.31)$$

Recall also that the Kähler potential for the complex structure sector is given by

$$K_{\text{cs}} = -\log \left(-i \int_{\mathcal{X}} \Omega \wedge \bar{\Omega} \right) . \quad (6.1.32)$$

In our model we obtain $K_{\text{cs}} = -3 \log (U + \bar{U})$ by setting $X^0 = 1$ and $d_{111} = 1$. Thus, in physical regime we have $v > 0$. Since the D-term potential (6.1.26) must be positive definite, we have $\text{Im} \mathcal{N} < 0$. Therefore, $d_{1\hat{1}\hat{1}} > 0$ follows. Substituting various preceding results in (6.1.26), the D-term potential reads

$$V_D = \frac{\delta}{v\tau^2} \left(g - \frac{r\tau}{3s} \right)^2 , \quad (6.1.33)$$

where δ is a positive constant. One can see that this potential depends on all the saxions in the model. The fluxes entering in V_D are related to the action of the twisted differential \mathcal{D} on the even $(2,1)$ forms. Such fluxes do not enter at all in the superpotential W that determines the F-term potential. However, there are Bianchi identities that mix $r_{\hat{\lambda}}$ and $f_{\hat{\lambda}\alpha}$ with NS-NS and Q -fluxes that might appear in W . In the model at hand the mixed BI constraints are

$$r \tilde{h}^{\lambda} + g \tilde{q}^{\lambda 1} = 0 , \quad r h_{\lambda} + g q_{\lambda}^1 = 0 , \quad (6.1.34)$$

for $\lambda = 0, 1$.

Uplift via D-term

Now we study the positive definite contribution from the derived D-term potential (6.1.33), where $r = f_{10}$, $g = f_{11}$, and δ is an unphysical positive constant which can be absorbed in

a redefinition of the fluxes. The superpotential leading to an additional F-term potential V_F is chosen to be

$$W = i\mathfrak{f}U + i\tilde{\mathfrak{f}}U^3 - ihS + iqT, \quad (6.1.35)$$

where we redefined $\mathfrak{f}_1 = -\mathfrak{f}$, $\tilde{\mathfrak{f}}^0 = \tilde{\mathfrak{f}}$, $h_0 = -h$ and $q_0^1 = -q$. After imposing the Bianchi identities (6.1.34), the D-term takes the form

$$V_D = \frac{\delta g^2}{\tau^2 v} \left(1 + \frac{q}{3h} \frac{\tau}{s}\right)^2. \quad (6.1.36)$$

By a suitable choice of δ , the total scalar potential $V = V_F + V_D$, admits a tachyon-free (stable) Minkowski minimum with axions fixed at

$$Re : \Theta = q\rho - hc = 0, \quad u = 0, \quad (6.1.37)$$

and the saxions take the form

$$s = \gamma_1 \frac{\mathfrak{f}^{3/2}}{h\tilde{\mathfrak{f}}^{1/2}}, \quad \tau = \gamma_2 \frac{\mathfrak{f}^{3/2}}{q\tilde{\mathfrak{f}}^{1/2}}, \quad v = \gamma_3 \left(\frac{\mathfrak{f}}{\tilde{\mathfrak{f}}}\right)^{1/2}. \quad (6.1.38)$$

The coefficient δ in (6.1.36) is given by

$$\delta g^2 = \gamma_4 \frac{hq\tilde{\mathfrak{f}}}{\mathfrak{f}}, \quad (6.1.39)$$

with numerical coefficients

$$\gamma_i = \{0.1545, 1.5761, 1.0318, 0.0044\}. \quad (6.1.40)$$

We can stay in the physical region with $\delta > 0$ and $\mathfrak{f}, \tilde{\mathfrak{f}}, h, q > 0$. The saxions are fixed in their perturbative regime for $\mathfrak{f} \gg \tilde{\mathfrak{f}}$, while $\tilde{\mathfrak{f}}, h, q$ are of order one. The normalized masses are given by

$$M_{\text{mod},i}^2 = \mu_i \frac{hq^3 \tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{9/2}} \frac{M_{\text{Pl}}^2}{4\pi}, \quad (6.1.41)$$

with prefactors

$$\mu_i = \{0.6986, 0.0152, 0.1318; 0.2594, 0.0524, 0\}. \quad (6.1.42)$$

Apparently there is one massless axion and the next lightest state is a saxion. The KK and string scales are given by

$$M_s^2 = 1.428 \frac{h^{1/2} q^{3/2} \tilde{f}}{\mathfrak{f}^3} M_{\text{Pl}}^2, \quad M_{\text{KK}}^2 = 0.008 \frac{q^2 \tilde{f}}{\mathfrak{f}^3} M_{\text{Pl}}^2. \quad (6.1.43)$$

The ratio of the KK and string scale is

$$\frac{M_s^2}{M_{\text{KK}}^2} = 178 \frac{h^{1/2}}{q^{1/2}}, \quad \frac{M_{\text{KK}}^2}{M_{\text{mod}}^2} = \frac{0.1}{\mu_i} \frac{1}{hq} \frac{\mathfrak{f}^{3/2}}{\tilde{f}^{3/2}}. \quad (6.1.44)$$

We see that $M_s > M_{\text{KK}}$ for $h > q$, and $M_{\text{KK}} > M_{\text{mod}}$ for $\mathfrak{f} \gg \tilde{f}$, hence in the perturbative regime the KK scale is parametrically heavier than the moduli mass scale. Since we have in addition one massless axion, that is a possible inflaton candidate, this model is a good starting point to study F-term axion monodromy inflation.

6.2 Axion monodromy inflation

In this section, we study the inflaton potentials we derived in the last section, by including the D-term generated by non-geometric fluxes. Recall that to guarantee the consistency of the effective field theory approach as well as to realize single field inflation, one has to stabilize the moduli such that the mass hierarchy $M_s > M_{\text{KK}} > M_{\text{inf}} > M_{\text{mod}} > H_{\text{inf}} > M_\theta$ is fulfilled. H_{inf} is the Hubble scale during inflation and $M_{\text{inf}} = V_{\text{inf}}^{\frac{1}{4}}$ is the mass scale of inflation.

6.2.1 Effective field theory approach

Recall that in section 6.1.2, we obtain one unstabilized and therefore massless axion via D-term uplift. As proposed in [134, 178] one can try to generate a parametrically small mass for this axion by turning on additional fluxes and scale the former fluxes by a parameter λ . Note that a good candidate for the extra flux is a P -flux [140], we construct our extended superpotential as¹

$$W = \lambda W_0 - ip S T U, \quad (6.2.1)$$

where W_0 takes the same form as in (6.1.35).

The new superpotential generates an F-term scalar potential in which the former terms scale with λ^2 . In the large λ limit, we approach to the former minimum as for (6.1.35).

¹Note that the full set of fluxes in W is not constrained by Bianchi identities.

The D-term potential is scaled to be

$$V_D = \lambda^2 \frac{(\delta_0 + \Delta\delta)g^2}{\tau^2 v} \left(1 + \frac{q}{3h} \frac{\tau}{s}\right)^2. \quad (6.2.2)$$

Here we have split δ into δ_0 given by the former value (6.1.39) plus a correction term $\Delta\delta$ to guarantee a Minkowski minimum also after including the P -flux.

When λ is large and thus we work in a $1/\lambda$ expansion, the leading order contribution to the shift in the uplift parameter turns out to be

$$\Delta\delta \sim -\frac{p\mathfrak{f}}{\lambda g^2}. \quad (6.2.3)$$

Assuming λ is sufficiently large, one can integrate out the heavy moduli and derive an effective potential for the former massless axion which is the orthogonal combination to Θ in (6.1.37). Since at the minimum $\Theta = 0$ we can take this axion to be $\theta = c$. Integrating out the heavy moduli, we obtain the effective quartic potential

$$V_{\text{eff}} = B_1 \theta^2 + B_2 \theta^4, \quad (6.2.4)$$

with

$$B_1 \sim \frac{\lambda p h^2 q^2 \tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{11/2}}, \quad B_2 \sim \frac{p^2 h^3 q \tilde{\mathfrak{f}}^{5/2}}{\mathfrak{f}^{13/2}}. \quad (6.2.5)$$

Since λ appear in B_1 , rather than B_2 , for sufficiently large λ , one can ensure that the quadratic term is dominant for θ of $\mathcal{O}(10)$, as favored by large field inflation. Furthermore, the ratios of mass scales turn out to be

$$\frac{M_{\text{KK}}^2}{M_{\text{mod}}^2} \sim \frac{\mathfrak{f}^{3/2}}{\lambda^2 h q \tilde{\mathfrak{f}}^{3/2}}, \quad \frac{M_{\text{mod}}^2}{M_\theta^2} \sim \frac{\lambda h q \tilde{\mathfrak{f}}}{p \mathfrak{f}^2}. \quad (6.2.6)$$

For large λ the inflaton mass becomes parametrically lighter than the mass of all the other moduli, which however are in danger of becoming heavier than the KK scale. Taking the product of the two mass ratios one gets

$$\frac{M_{\text{KK}}^2}{M_{\text{mod}}^2} \frac{M_{\text{mod}}^2}{M_\theta^2} \sim \frac{1}{\lambda p \mathfrak{f}^{1/2} \tilde{\mathfrak{f}}^{1/2}}. \quad (6.2.7)$$

It is clear that as long as all these fluxes are positive integers and λ to be large, it is in principle impossible to have both mass ratios to fulfill that $M_{\text{KK}}^2 \gg M_\theta^2$. As this stage, we reconsider the model with rational flux values.

One potential loophole in this no-go result is the assumption that all fluxes are integer quantized. In fact, as also realized in [184], the prepotential for the complex structure moduli in the large complex structure limit is subject to perturbative and non-perturbative

corrections, which take the general form (see for instance [185])²

$$\tilde{F} = F + \frac{1}{2}a_{ij}X^iX^j + b_iX^iX^0 + \frac{1}{2}i\gamma(X^0)^2 + F_{\text{inst.}}, \quad (6.2.8)$$

with the usual cubic term $F = \frac{1}{6}d_{ijk}X^iX^jX^k/X^0$. Here, the constants a_{ij} and b_i are rational numbers, while γ is real. From the point of view of the mirror dual threefold \hat{M} , they are determined as

$$\begin{aligned} a_{ij} &= -\frac{1}{2} \int_{\hat{M}} \hat{\omega}_i \wedge \hat{\omega}_j \wedge \hat{\omega}_j, \quad b_i = \frac{1}{24} \int_{\hat{M}} c_2(\hat{M}) \wedge \hat{\omega}_i, \quad \text{mod } \mathbb{Z} \\ i\gamma &= \frac{1}{(2\pi i)^3} \chi(\hat{M}) \zeta(3), \end{aligned} \quad (6.2.9)$$

with the second Chern class $c_2(\hat{M})$, the Euler number of the internal space $\chi(\hat{M})$ and a basis of harmonic $(1,1)$ -forms $\hat{\omega}_i$. These constants can be smaller than one, but not arbitrarily small. Note that when evaluating the superpotential (3.2.28), the corrections a_{ij} and b_i can be incorporated by the following shifts in the fluxes $g_\Lambda \in \{f_\Lambda, f_{\Lambda a}, q_\Lambda^\alpha\}$

$$g_0 = g_0 - b_i \tilde{g}^i, \quad g_i = g_i - a_{ij} \tilde{g}^j - b_i \tilde{g}^0. \quad (6.2.10)$$

Recall that the purely imaginary contribution $i\gamma$ corresponds to α' -corrections to the Kähler potential for the Kähler moduli in a mirror-dual setting. In the large complex-structure regime we are employing here, these corrections can be neglected. Similarly, in this regime also the non-perturbative corrections $F_{\text{inst.}}$ are negligible. To summarize, the polynomial corrections to the prepotential can be incorporated by a rational shift in the fluxes. This at least motivates the numerical approach to be adopted in the following section³.

6.2.2 Numerical analysis of inflation

Instead of pursuing an effective approach, as in the previous subsection, we now follow an exact, though numerical, approach to analyze the same model. In practice we choose initial (phenomenologically motivated) values of the fluxes, compute the exact scalar potentials in terms of all moduli fields and then study numerically for stable Minkowski minima. We are particularly interested in determining whether there exists a choice of (rational) fluxes so that we can concretely realize the mass hierarchy (6.0.1).

In Figure 6.2, we illustrate, for a certain choice of fluxes, the behavior of some relevant mass ratios according to the scaling parameter λ . Fluxes are chosen rational with values

²Note that the terminology of perturbative and non-perturbative corrections is actually taken from the mirror dual side, where the complex structure moduli are exchanged with the Kähler moduli.

³Let us mention that in other recent works [46, 48] on de Sitter vacua of string theory, the fluxes were also chosen to be rational.

$h = 1/220$, $\tilde{f} = 1/1810$, $f = 6/49$, $q = 1/8$, $g = 1/10$ and $p = 1/10000$. As it is shown, for

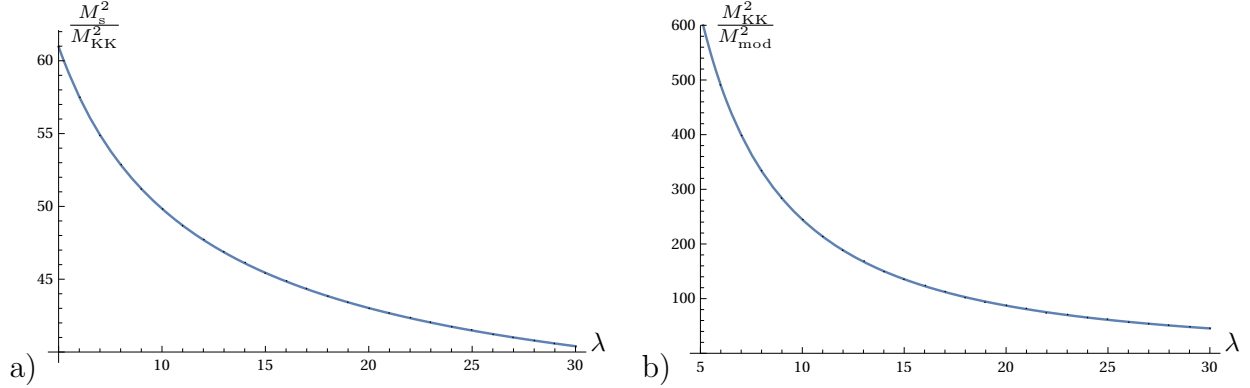


Figure 6.2: Ratios of relevant mass scales

all values of λ , the KK and string mass are separated by a factor of $\mathcal{O}(10)$. Moreover, the heaviest moduli mass is lower than the KK scale by a factor of $\mathcal{O}(10^2)$ for small λ . Even for values of $\lambda \sim 30$, the heaviest moduli mass is lower than the KK scale by a factor of $\mathcal{O}(10)$. Thus, we have overall control over the mass hierarchy

$$M_{Pl} > M_s > M_{KK} > M_{mod} . \quad (6.2.11)$$

The axions are fixed at

$$\Theta = \theta = u = 0 , \quad (6.2.12)$$

The saxions versus λ is shown in Figure 6.3 for the same fluxes' value as in Figure 6.2.

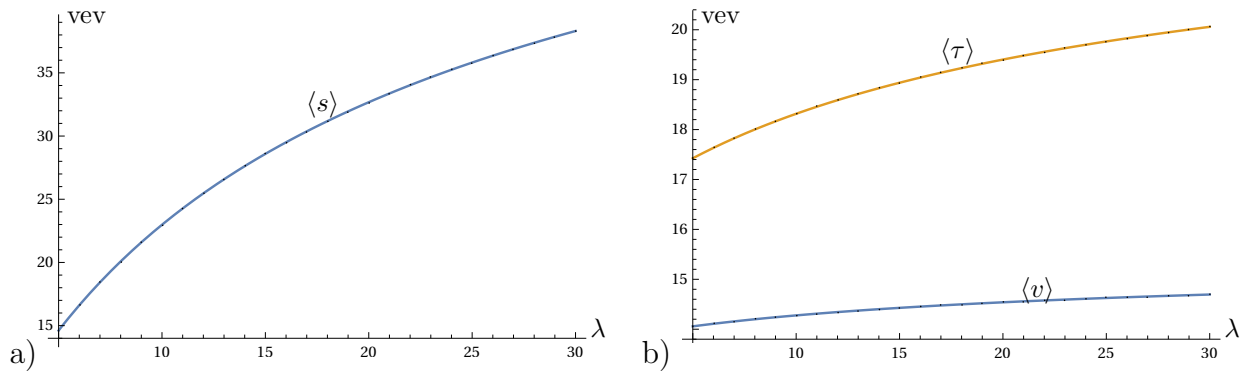


Figure 6.3: Vevs of the saxionic moduli for s , τ and v .

We observe that as λ increases the saxionic vevs increase so that we can trust the perturbative expansion for all λ . Let us mention that for $\lambda < 5$ tachyons appear in the spectrum that are not shown in Figure 6.2. Finally, for all λ the lightest state is related to

the axion c and its mass is smaller than the next heavier state by a factor of $\mathcal{O}(10^2)$. In the following will consider c as the inflaton candidate.

Inflation model with $\lambda = 10$

Next, for the values of the fluxes shown above and choosing $\lambda = 10$, we consider the backreaction [186] of the slowly rolling light axion $\theta = c$. The main task is to solve the extremum conditions $\partial_i V = 0$ to obtain the saxions as functions of θ . Fixing all the heavy moduli at the minimum, the effective scalar potential turns out to be

$$V_{\text{eff}}(\theta) \approx B_1 \theta^2 + B_2 \theta^4, \quad (6.2.13)$$

where $B \cdot 10^{14} = \{2.8711, 6.8314 \cdot 10^{-6}\}$. Thus, the quartic term is suppressed by a factor of $\mathcal{O}(10^{-6})$, and the effective scalar potential for sufficiently small θ has a quadratic behavior. To have a Minkowski vacuum we must have $\delta \cdot 10^7 = 6.0647$. Figure 6.4 illustrates that the scalar potential including the backreaction as well as the effective scalar potential in (6.2.13). We observe that near $c = 0$ both potentials match, while the backreaction modifies the shape of the scalar potential for larger values of the inflaton θ , producing a plateau-like behavior. We gave the backreaction and quadratic potential in the units $\frac{M_{\text{Pl}}^4}{4\pi}$ given by eq. (6.2.13) for $h = 1/220$, $\tilde{f} = 1/1810$, $f = 6/49$, $q = 1/8$, $g = 1/10$, $p = 1/10000$ and $\lambda = 10$. In order to compute the cosmological quantities n_s , ϵ , η and N_e , we first

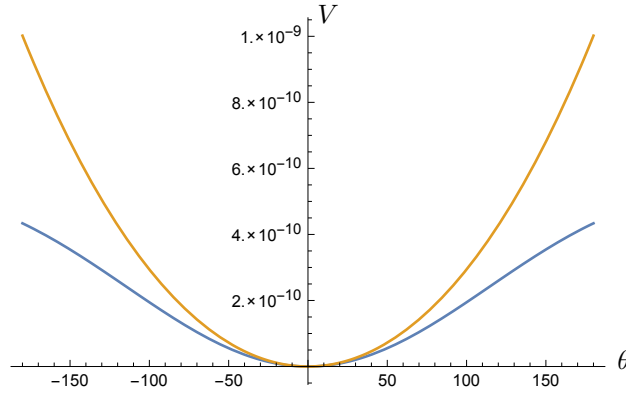


Figure 6.4: Backreaction (blue) and quadratic potential (brown).

calculate the slow-roll parameters ϵ and η as in [178]. Recall that for the Lagrangian $\mathcal{L} = \frac{1}{2}f(\theta)^2(\partial\theta)^2 + V(\theta)$ the slow-roll parameters are given by

$$\epsilon = \frac{1}{2f} \left(\frac{\partial V}{V} \right)^2, \quad \eta = \frac{\partial^2 V}{fV} - \frac{\partial f \partial V}{2f^2 V}. \quad (6.2.14)$$

The end of inflation is determined by the point of the moduli space in which the slow-roll conditions are violated, i.e. $\epsilon \sim 1$. The starting point for the inflationary trajectory is

chosen in such a way that $n_s = 0.9667 \pm 0.004$ [6]. The e-foldings as well as the tensor-to-scalar ratio (evaluated at the pivot scale θ^*) are then derived from

$$r = 16\epsilon_*, \quad N_e = \int_{\theta_{\text{end}}}^{\theta^*} d\theta \frac{fV}{\partial V}. \quad (6.2.15)$$

Recall that the value of the amplitude of the scalar power spectrum reported experimentally is $\mathcal{P} = (2.142 \pm 0.049) \cdot 10^{-9}$, moreover it is determined from the Hubble scale and ϵ at the pivot scale by

$$\mathcal{P} \sim \frac{H_{\text{inf}}^2}{8\pi^2 \epsilon_* M_{\text{Pl}}^2}. \quad (6.2.16)$$

From which one derives the Hubble scale during inflation. For the choice of fluxes mentioned above, we get the inflationary parameters in Table 6.1. For $9.44 < \theta < 104$ one

Parameter	Value
Δc	$93 M_{\text{Pl}}$
N_e	61
r	0.0980
n_s	0.9667
\mathcal{P}	$2.14 \cdot 10^{-9}$
M_s	$1.04 \cdot 10^{17} \text{ GeV}$
M_{KK}	$1.49 \cdot 10^{16} \text{ GeV}$
M_{inf}	$4.89 \cdot 10^{15} \text{ GeV}$
M_{mod}	$\{11.99, 4.81, 2.38, 6.81, 2.47\} \cdot 10^{14} \text{ GeV}$
H_{inf}	$7.82 \cdot 10^{13} \text{ GeV}$
M_θ	$1.70 \cdot 10^{13} \text{ GeV}$

Table 6.1: Summary of inflationary parameters for $\lambda = 10$.

collects 60 e-foldings for the reported spectral index n_s . Read from Table 6.1, we obtain the hierarchy of mass scales

$$M_{\text{Pl}} > M_s > M_{\text{KK}} > M_{\text{inf}} > M_{\text{mod}} > H_{\text{inf}} > M_\theta, \quad (6.2.17)$$

with all individual scales as expected. However the value for the tensor-to-scalar ratio lies on the boundary of being ruled out experimentally and is a bit smaller than the value for quadratic inflation.

This numerical example shows that by allowing rational values of the fluxes, in particular those smaller than one, it is in principle possible to freeze all moduli such that the above desired hierarchy of mass scales is realized. Of course for a concrete Calabi-Yau manifold the parameters for the polynomial terms in the prepotential (6.2.8) are fixed and therefore

the admissible fluxes are more constrained than assumed in our phenomenological study. In particular, non-vanishing fluxes can not be smaller than $|1/24|$ and the flux \tilde{f} according to (6.2.10) would still be an integer.

Inflation model with $\lambda = 5$

Here we consider the same model but choose the limit case with $\lambda = 5$. Recall that for $\lambda < 5$, tachyons appears on the spectrum. As in the previous case, the lightest state is axionic and related to $\theta = c$.

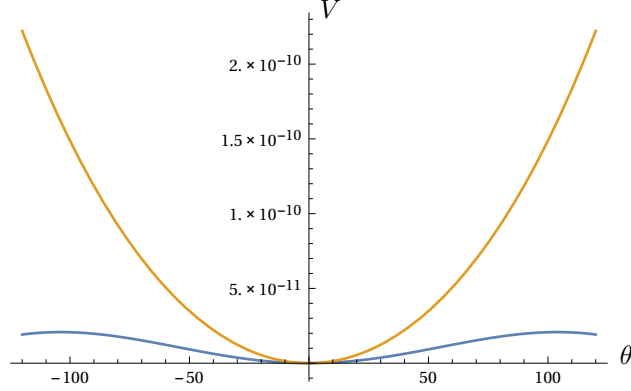


Figure 6.5: Backreaction (blue) and quadratic potential (brown).

For this limit situation we have, as shown in Figure 6.2, a greater separation between the KK scale and the string scale, while the vevs for the moduli are kept in the perturbative regime. The effective scalar potential for $\lambda = 5$ has the form (6.2.13) with coefficients $B \cdot 10^{14} = \{1.3607, 1.2675 \cdot 10^{-5}\}$, so that it effectively behaves as a quadratic potential near the origin (see Figure 6.5). In this case a Minkowski vacuum is obtained by taking $\delta \cdot 10^7 = 4.2004$. We illustrate the backreaction and quadratic potential in units $\frac{M_{\text{Pl}}^4}{4\pi}$ given by eq. (6.2.13) for $h = 1/220$, $\tilde{f} = 1/1810$, $f = 6/49$, $q = 1/8$, $g = 1/10$, $p = 1/10000$ and $\lambda = 5$.

As expected, for lower values of λ the flattening effect of the backreaction becomes more important. In Table 6.2 we show the relevant cosmological parameters for $\lambda = 5$. We find a similar pattern as in the model presented in section 6.2.2, but now the number of e-foldings is fairly large, while the tensor-to-scalar ratio is almost as low as for the Starobinsky model. By decreasing λ , the model changes from quadratic to plateau-like inflation which has also been observed in [178].

Parameter	Value
Δc	$86 M_{\text{Pl}}$
N_e	125
r	0.007
n_s	0.9667
\mathcal{P}	$2.14 \cdot 10^{-9}$
M_s	$1.37 \cdot 10^{17} \text{ GeV}$
M_{KK}	$1.76 \cdot 10^{16} \text{ GeV}$
M_{inf}	$2.74 \cdot 10^{15} \text{ GeV}$
M_{mod}	$\{7.91, 3.11, 1.65; 6.68, 2.12\} \cdot 10^{14} \text{ GeV}$
H_{inf}	$2.08 \cdot 10^{13} \text{ GeV}$
M_θ	$4.69 \cdot 10^{12} \text{ GeV}$

 Table 6.2: Summary of inflationary parameters for $\lambda = 5$.

6.3 Summary and discussion

In this chapter, we studied the string phenomenology from type IIB orientifold compactifications with geometric and non-geometric fluxes turned on. We focused especially on the aspect of de Sitter uplift mechanisms and the following axion monodromy inflation.

We considered two possible energy sources contributing a positive semi-definite term to the scalar potential, namely an $\overline{\text{D3}}$ -brane and a D-term induced by geometric and non-geometric fluxes for non-zero $h_+^{2,1}$. Both approaches led to new de Sitter and Minkowski minima of flux-scaling type, rather than uplift initial flux-scaling minima.

We explored to what extent the uplifted models could serve as starting points for the realization of axion monodromy inflation with a parametrically controlled hierarchy of induced mass scales. We found that the required hierarchy among the KK scale, the moduli mass scale and the axion mass scale was not achieved when we insisted on integer fluxes. Recalling that the perturbative corrections to the prepotential of the complex structure moduli effectively lead to a redefinition of the fluxes, we performed a numerical model search admitting also rational values of all fluxes. In this way we presented examples where all the desired properties could be obtained.

However, a concrete Calabi-Yau compactification models which give rise to the required flux rational values still calls for further investigation. So far, such a manifold with an orientifold projection has not been specified. Therefore, it has not been established conclusively that the considered vacua of four-dimensional gauged supergravity do uplift to full solutions of ten-dimensional string theory.

Part III

Conclusions and Outlook

Conclusions and Outlook

In this thesis we have studied non-geometric backgrounds, as well as their applications in deformations of geometry and string phenomenology. We conclude the thesis with a summary of the projects and an outlook on further investigations. Since at the end of each chapter in part II we have already provided a detailed summary of each project, we will focus on their interplay, general conclusion and outlook.

7.1 Summary of research projects

- Non-geometry in heterotic double field theory
- Non-associative deformations of geometry
- String phenomenology with non-geometries

Non-geometry in heterotic double field theory

In the recent study of string phenomenology and flux compactification, non-geometric fluxes are often used together with geometric ones, considered as T-duals to each other. This can be given with a natural description in double field theory (DFT), which initially constructed as a generalization of supergravity with T-duality manifested into an $O(D, D)$ global symmetry. It provides a convenient way to study the non-geometric backgrounds as the T-duals of the standard geometric ones.

Here, we worked on getting a better understanding of the heterotic generalization of DFT. We found non-geometric backgrounds as the T-dual of constant gauge flux backgrounds, analogous to the study of the Kalb–Ramond B-field. In addition, we studied the T-duality mapping in terms of the differential geometry of a corresponding Lie algebroid in generalized geometry framework, and how the gauge field takes part in it. We showed that the resulting field redefinitions from the Lie algebroid anchor mapping is consistent with those from the heterotic Buscher transformations. In particular, the α' corrections are naturally incorporated within the gauge field terms. With the understanding we gained

from heterotic DFT, we found that the constant non-geometric gauge J -flux background of the $E_8 \times E_8$ heterotic string can be considered as the S-dual of a Type I' background with a $D8$ -brane intersecting the $O8$ -plane at an angle. Moreover, we showed that the T-dual of the heterotic supergravity action (the one corresponding to a non-geometric frame) can be derived from the $O(D, D + n)$ induced Lie algebroid anchor. We expect that the whole action including the fermionic terms is governed by the objects in the differential geometry of the Lie algebroid. This includes e.g. the kinetic terms for the gravitinos and gluinos, which involve a spin-connection. Furthermore, the gravitational Chern-Simons terms follow similar rules as the gauge field terms.

Moreover, we gave explicit expressions of all the possible fluxes, including gauge fluxes in heterotic DFT framework. We studied the analogous structure in $SL(5)$ theory.

Non-associative deformations of geometry

The link between string theory and non-commutative geometry has been established since Seiberg and Witten shed light on it. For example, the effective theory for an open string moving on a D-brane becomes a non-commutative gauge theory if a constant flux is switched on. For open strings, it has been shown that in the background of a non-constant two-form the coordinates are non-commutative and non-associative. In closed string theory, one is necessarily also dealing with gravity and the possible target space deformations. These turn out to be deformations by a Tri-product structure which can be considered as non-associative deformations of geometry. From a CFT point of view such a structure is absent on-shell, but off-shell there is still room for it. For closed strings, the two-products are different from those of open string. In two-dimensional CFT, for three-point functions, we can define an orientation connecting them and thus we will expect a similar non-commutativity arising as for open strings. This has been studied for closed strings and the correlator of three tachyon vertex operators was computed for both the geometric H-flux and the non-geometric R-flux backgrounds.

Moreover, we analyzed how the structure derived from the CFT perspective carried over to the recently discussed (non-)geometric framework of DFT, where the geometric fluxes and the non-geometric fluxes are well-defined and unified into a doubled flux F^{ABC} . We computed the generic Tri-products with the generic functions being scalars. We showed that up to leading order the Tri-products of modified fluxes give boundary terms when the DFT equations of motion are satisfied. However, flux F^{ABC} differs from what we have at the CFT point of view, as the non-geometric R-flux is a three-vector. The non-associativity is annihilated if the strong constraint is applied. Furthermore, in heterotic DFT, we expect it to be quite similar, while in exceptional field theory (EFT) with S-duality incorporated, its properties need further investigation.

String phenomenology with non-geometries

In the flux compactification project, we tried to construct de Sitter vacua and study the large field inflation model therein. We implemented the common mechanism to uplift AdS vacua to de Sitter vacua and preserve stability by introducing an $\overline{\text{D3}}$ -brane at a warped throat as in the KKLT scenario. In orientifold compactifications of type IIB on CYs, non-vanishing fluxes generically induce superpotentials. Thus, the closed string moduli, namely the axio-dilation as well as the complex structure and Kähler moduli, will be stabilized by fluxes.

We studied type IIB orientifold compactifications with geometric and non-geometric fluxes turned on. We constructed a sequence of AdS vacua via moduli stabilization from the reduced F-term scalar potentials. By implementing an $\overline{\text{D3}}$ -brane in a warped throat as in the KKLT scenario, we analyzed the extra positive contributions to the scalar potential. We found tachyon-free non-supersymmetric Minkowski and de Sitter vacua. An analytical method to uplift Minkowski to de Sitter vacua by perturbing around the original vacua was constructed.

As a second uplift approach, by setting $h_+^{2,1} > 0$, we included the abelian gauge fields coming from the dimensional reduction of the R-R four-form on an orientifold even three-cycle of CY. This setting introduces new contributions to the scalar potential from the D-term. We showed that it admits tachyon-free Minkowski/de Sitter vacua. By introducing an extra P -flux term (which is considered to be the S-dual of Q -flux) we obtained a de Sitter vacuum with a good inflaton candidate. This procedure provides a flux-scaling scenario.

On the axion inflation aspects, we derived the axion potential from the F-term scalar potential. Based on the de Sitter vacua we found, where the lightest state is given by an axion moduli, we obtained axion inflation models in which the mass hierarchy is fully satisfied. However, rational shifts of the flux values need to be admitted. It is interesting to study the precise allowed rational shifts from the perturbative corrections of the prepotential which highly depend on the geometry of the compactification manifold.

7.2 Outlook

In this section, we will present some further open questions in the above fields. For example, in general, it would be interesting to study further in orientifold compactification and its applications, as well as to extend this work to U-dualities in EFT.

Orientifold compactifications and its applications

In our investigation of de Sitter uplifts, studying the perturbative and non-perturbative corrections to the prepotential and reduced scalar potential for CY manifolds would give us more details about the allowed rational shifts for fluxes, either analytically or by examples.

It is interesting to follow this line in order to check whether well consistent de Sitter vacua are achievable. In addition, further study of the KKLT scenario in the $\overline{\text{D3}}$ -brane uplift example and possible axion inflation models are also appealing. Furthermore, studying the supersymmetry breaking soft terms from (non-)geometric fluxes for particle physics phenomenology is also an interesting topic to explore.

Moreover, it has been shown that α' corrections can be neglected in the ‘large volume limit’. It is possible that the appearance of non-geometric fluxes does not ruin the standard large volume limit, but to be more convincing it deserves a more analytical study. It might be interesting to solve the equation of motion including contributions from the non-geometric frame, with DFT action, to explicitly show the non-geometric modifications.

U-duality, M-theory and Exceptional Field Theory

In M-theory, the global symmetry is U-duality instead of T-duality with internal exceptional gauge groups. Whether the U-dual action can be derived from the differential geometry of a Lie algebroid, whose anchor corresponds to the exceptional gauge groups, is under investigation. We expect that a Lie algebroid anchor mapping between the exceptional tangent bundles can provide U-duality manifested field redefinitions. So far, we have been successful in $E_4 = SL(5)$ theory (which is considered as one of the toy models of EFT). We obtained the Lie algebroid anchor for $SL(5)$ theory and also confirmed that this is consistent with the $SL(5)$ field redefinitions. Consequently, the redefined action can be given by the differential geometry of a Lie algebroid, whose anchor corresponds to the $SL(5)$ transformation. As an ongoing project, we expect this mapping can be generalized to certain higher-dimensional EFTs. However, this is a non-trivial task. For its applications, recall that in EFT formulations, the fluxes transform under different rules with S-duality manifested. This feature makes it interesting to study the non-associativity in EFT to gain new understanding for the connection of geometry and M-theory.

String embeddings of non-geometry

Although we have presented the source of non-geometric backgrounds from T-duality/U-duality in DFT/EFT, the embeddings of non-geometric fields and gauge vectors in string theory need more clarifications. In some sense, it is analogous to the study of Kaluza-Klein reductions in the standard geometric frame. It might be interesting to study double sigma models and Scherk-Schwarz reductions for possible embedding of non-geometries.

In summary, in the current era of high energy physics, new experimental data on the completion of Standard Model and early universe are obtained or expected. This highly motivates the study of string theory and string phenomenology. We observed and expect that non-geometric backgrounds can also play a role in this exploration.



K tri-product

In this appendix we discuss how to treat terms which involve for instance a product of K functions. Clearly, e.g. for $K = 4$ this is not defined by an iteration of the tri-product (5.3.32). From the analysis of multiple tachyon scattering amplitudes in CFT, in [113] a proposal was made, how to deform the product of K functions. Analogously, at leading order in (\mathcal{DF}_{ABC}) (or $(\check{\mathcal{DF}}_{ABC})$) we now define the K -fold tri-product as

$$(f_1 \Delta_K f_2 \Delta_K \dots \Delta_K f_K)(X) \stackrel{\text{def}}{=} \exp\left(\frac{\ell_s^4}{6} \mathcal{F}_{ABC} \sum_{1 \leq a < b < c \leq K} \mathcal{D}_{X_a}^A \mathcal{D}_{X_b}^B \mathcal{D}_{X_c}^C\right) f_1(X_1) f_2(X_2) \dots f_K(X_K) \Big|_X. \quad (\text{A.1})$$

Below we prove the remarkable feature that for each K all terms beyond leading order give a total derivative under the internal integral, i.e.

$$\int dX e^{-2d} f_1 \Delta_K f_2 \Delta_K \dots \Delta_K f_K = \int dX e^{-2d} f_1 f_2 \dots f_K. \quad (\text{A.2})$$

Moreover, this K tri-product has the property

$$f_1 \Delta_K \dots \Delta_K 1 = f_1 \Delta_{K-1} \dots \Delta_{K-1} f_{K-1} \quad (\text{A.3})$$

which suggests to define $f_1 \Delta_2 f_2 = f_1 \cdot f_2$, i.e. the two tri-product is the ordinary multiplication of functions.

Note that the total derivative property does *not* hold for a similar definition of an K star-product

$$(f_1 \star_K f_2 \star_K \dots \star_K f_K)(X) \stackrel{\text{def}}{=} \exp\left(i \frac{\ell_s^2}{2} \theta^{IJ} \sum_{1 \leq a < b \leq K} \partial_I^{X_a} \partial_J^{X_b}\right) f_1(X_1) f_2(X_2) \dots f_K(X_K) \Big|_X, \quad (\text{A.4})$$

This is why for the open string case, the non-commutativity of the underlying spacetime has a non-trivial effect on the action.

Proof

Here we present the proof that at leading order in \mathcal{DF}_{ABC} the K tri-product (A.1) gives a total derivative under the integral, i.e.

$$\int dX e^{-2d} f_1 \Delta_K \dots \Delta_K f_K = \int dX e^{-2d} f_1 \dots f_K. \quad (\text{A.5})$$

We first consider just the order ℓ_s^4 term, which is given by

$$\frac{\ell_s^4}{6} \mathcal{F}_{ABC} \sum_{1 \leq a < b < c \leq K} \mathcal{D}_{X_a}^A \mathcal{D}_{X_b}^B \mathcal{D}_{X_c}^C \left(f_1(X_1) f_2(X_2) \dots f_K(X_K) \right) \Big|_X.$$

Inspection reveals, that the $\binom{K}{3}$ terms can be grouped together as

$$\begin{aligned} & \mathcal{D}^A(f_1) \mathcal{D}^B f_2 \mathcal{D}^C(f_3 \dots f_K) \\ & + \mathcal{D}^A(f_1 f_2) \mathcal{D}^B f_3 \mathcal{D}^C(f_4 \dots f_K) \\ & + \mathcal{D}^A(f_1 f_2 f_3) \mathcal{D}^B f_4 \mathcal{D}^C(f_5 \dots f_K) \\ & + \dots \\ & + \mathcal{D}^A(f_1 \dots f_{K-2}) \mathcal{D}^B f_{K-1} \mathcal{D}^C(f_K). \end{aligned} \quad (\text{A.6})$$

Note that the sum fixes the order of the derivatives and the number of terms is correct, since

$$\binom{K}{3} = 1 \cdot (K-2) + 2 \cdot (K-3) + \dots + (K-2) \cdot 1. \quad (\text{A.7})$$

As one can see, the K tri-product splits into $K-2$ three tri-products and therefore shares its properties under an integral. The higher order terms follow immediately by iteration. This is owed to the fact that, in the derivation of the total derivative property, only first three derivatives are relevant.

B

The heterotic Buscher rules

Using the implementation of T-duality in heterotic DFT, one can now quite generally derive the heterotic Buscher rules from the conjugation of the generalized metric with the corresponding T-duality matrix. Carrying out this procedure for a T-duality in the x^θ direction, we get precisely the α' corrected heterotic Buscher rules presented in [143]

$$\begin{aligned} G'_{\theta\theta} &= \frac{G_{\theta\theta}}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)^2}, \\ G'_{\theta i} &= -\frac{G_{\theta\theta} B_{\theta i} + \frac{\alpha'}{2} G_{\theta i} A_\theta^2 - \frac{\alpha'}{2} G_{\theta\theta} A_\theta A_i}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)^2}, \\ G'_{ij} &= G_{ij} - \frac{G_{\theta i} G_{\theta j} - B_{\theta i} B_{\theta j}}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)} \\ &\quad - \frac{1}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)^2} \left(G_{\theta\theta} \left[\frac{\alpha'}{2} B_{\theta j} A_\theta A_i + \frac{\alpha'}{2} B_{\theta i} A_\theta A_j - \frac{\alpha'^2}{4} A_\theta A_i A_\theta A_j \right] \right. \\ &\quad \left. + \frac{\alpha'}{2} A_\theta^2 \left[(G_{\theta i} - B_{\theta i})(G_{\theta j} - B_{\theta j}) + \frac{\alpha'}{2} (G_{\theta i} A_\theta A_j + G_{\theta j} A_\theta A_i) \right] \right), \quad (\text{B.1}) \\ B'_{\theta i} &= -\frac{G_{\theta i} + \frac{\alpha'}{2} A_\theta A_i}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)}, \\ B'_{ij} &= B_{ij} - \frac{(G_{\theta i} + \frac{\alpha'}{2} A_\theta A_i) B_{\theta j} - (G_{\theta j} + \frac{\alpha'}{2} A_\theta A_j) B_{\theta i}}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)}, \\ A'^\alpha_\theta &= -\frac{A_\theta^\alpha}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)}, \\ A'^\alpha_i &= A_i^\alpha - A_\theta^\alpha \frac{G_{\theta i} - B_{\theta i} + \frac{\alpha'}{2} A_\theta A_i}{\left(G_{\theta\theta} + \frac{\alpha'}{2} A_\theta^2\right)}, \end{aligned}$$

where e.g. $A_\theta A_i = A_\theta^\alpha A_{i\alpha}$. Here the metric and the Kalb-Ramond field have dimension $[l]^0$ and the gauge field $[l]^{-1}$.



Non-holonomic fluxes for heterotic DFT

In this appendix we present the explicit expressions of the fluxes in a non-holonomic basis. From the generalized vielbein E_A^M and the dilation d one can build the generalized fluxes

$$\begin{aligned}\mathcal{F}_{ABC} &= E_{CM}\mathcal{L}_{E_A}E_B^M = \Omega_{ABC} + \Omega_{CAB} - \Omega_{BAC}, \\ \mathcal{F}_A &= -e^{2d}\mathcal{L}_{E_A}e^{-2d} = -\partial_M E_A^M + 2D_A d.\end{aligned}\tag{C.1}$$

The generalized derivative $D_A = E_A^M D_M$ takes the form

$$\begin{aligned}\tilde{D}^a &= \tilde{\partial}^a + \tilde{C}^{am}C_{mn}\tilde{\partial}^n - \tilde{C}^{am}\partial_m - \tilde{A}^{a\gamma}\partial_\gamma, \\ D_a &= \partial_a - C_{am}\tilde{\partial}^m - A_a^\gamma\partial_\gamma, \\ D_\alpha &= \partial_\alpha + A_{m\alpha}\tilde{\partial}^m + \tilde{A}^m{}_\alpha\partial_m.\end{aligned}\tag{C.2}$$

As in section 3, we present the geometric fluxes for the physically relevant case of $\tilde{A}^a{}_\alpha = 0$ and the non-geometric fluxes for $A_a{}^\alpha = 0$. From the flux definition (C.1) we obtain the geometric fluxes ¹

$$\begin{aligned}H_{abc} &= -3(D_{[a}B_{bc]} - D_{[a}A_{b\gamma}A_{c]}^\gamma + f_{[ab}^m C_{c]m} - C_{[am}C_{bn}\tilde{f}_{c]}^{mn} - A_{[a}^\beta\partial_\beta e_{b]}^i C_{c]i}), \\ F_{ab}^c &= f_{ab}^c - \tilde{D}^c B_{ab} + \tilde{D}^c A_{[a\gamma}A_{b]}^\gamma + 2C_{[am}\tilde{f}_{b]}^{mc} - 2D_{[a}\beta^{cm}A_{b]\gamma}A_m^\gamma - 2\beta^{cm}D_{[a}C_{mb]} \\ &\quad + 3\beta^{cm}(f_{[ma}^n C_{b]n} - C_{[mn}C_{ap}\tilde{f}_{b]}^{np}) - 2\beta^{cm}(C_{mi}A_{[a}^\beta\partial_\beta e_{b]}^i + C_{[ai}A_{b]}^\beta\partial_\beta e_m^i) \\ &\quad - 2A_{[a}^\beta\partial_\beta e_{b]}^i e^c{}_i,\end{aligned}\tag{C.3}$$

¹Note that the derivative D^i, D_i and D_α will also be simplified.

and for $A_a{}^\alpha = 0$ the non-geometric fluxes read

$$\begin{aligned}
 Q_c{}^{ab} &= -D_c\beta^{ab} + D_c\tilde{A}^{[a\gamma}\tilde{A}^{b]}\gamma - 2\tilde{D}^{[a}B_{cn}\tilde{C}^{b]n} - \tilde{C}^{[am}\tilde{C}^{b]n}D_cB_{mn} + \tilde{f}_c{}^{ab} + 2\tilde{C}^{[am}\tilde{f}^{b]}_{mc} \\
 &\quad + 2B_{cm}\tilde{C}^{[an}\tilde{f}_n{}^{b]m} + 2\tilde{C}^{[am}B_{mn}\tilde{f}_c{}^{nb]} - 3\tilde{C}^{am}\tilde{C}^{bn}(B_{[mp}f_{nc]}^p - B_{[mp}C_{nq}\tilde{f}_{c]}^{pq}) \\
 &\quad + 2(B_{cm}\tilde{C}^{[an}\tilde{A}^{b]}\gamma\partial_\gamma e_n{}^i e_m{}^i + \tilde{A}^{[a\gamma}\partial_\gamma e_c{}^i e^{b]}_i - \tilde{C}^{[am}B_{mi}\tilde{A}^{b]}\gamma\partial_\gamma e_c{}^i), \\
 R^{abc} &= -3\tilde{D}^{[a}\beta^{bc]} + 3\tilde{D}^{[a}\tilde{A}^{b\gamma}\tilde{A}^{c]}\gamma + 3\tilde{C}^{[am}\tilde{D}^bB_{mn}\tilde{C}^{c]n} + 6\tilde{C}^{[am}\tilde{C}^{bn}B_{[mp}\tilde{f}_{n]}^{pc]} \\
 &\quad + 3\tilde{C}^{am}\tilde{C}^{bn}\tilde{C}^{cp}(B_{[mq}f_{np]}^q - B_{[mq}B_{nl}\tilde{f}_{p]}^{ql}) + 3(\tilde{C}^{[am}\tilde{C}^{bn}f_{mn}^{c]} - \tilde{C}^{[am}f_m{}^{bc]}) \\
 &\quad + 3(\tilde{C}^{[am}\tilde{C}^{bn}B_{ni}\tilde{A}^{c]}\gamma\partial_\gamma e_m{}^i - 2\tilde{C}^{[am}\tilde{A}^{b\gamma}\partial_\gamma e_m{}^i e^{c]}_i).
 \end{aligned} \tag{C.4}$$

For $A_i{}^\alpha = \tilde{A}^i{}_\alpha = 0$, these expressions coincide with the ones derived in [66] and [67]. Similarly, the fluxes \mathcal{F}_A can be expanded as

$$\begin{aligned}
 F_a &= -\partial_m e_a{}^m + \tilde{\partial}^m C_{am} + \partial_\alpha A_a{}^\alpha + 2D_a d, \\
 F^a &= \partial_m \tilde{C}^{am} - \tilde{\partial}^m e^a{}_m - \tilde{\partial}^m(\tilde{C}^{an}C_{nm}) + \partial_\alpha \tilde{A}^{a\alpha} + 2\tilde{D}^a d.
 \end{aligned} \tag{C.5}$$

Due to the extra gauge coordinates in heterotic DFT, we also have the gauge fluxes $G_{\alpha ab}$, $J^c{}_{ab}$ and $\tilde{G}_\alpha{}^{ab}$. For $\tilde{A}^i{}_\alpha = \beta = 0$ they become

$$\begin{aligned}
 G_{\alpha ab} &= -D_\alpha B_{ab} + D_\alpha A_{[a}{}^\gamma A_{b]}\gamma - 2D_{[a}A_{b]}\alpha + A_{\alpha m}f^m{}_{ab} + 2C_{[am}A_{n\alpha}\tilde{f}_{b]}^{mn}, \\
 &\quad + 2(C_{[ai}\partial_\alpha e_{b]}^i - A_{\alpha i}A_{[a}{}^\gamma \partial_\gamma e_{b]}^i), \\
 J^c{}_{ab} &= \tilde{\partial}^c A_{b\alpha} + A_{m\alpha}\tilde{f}_b{}^{cm} + \partial_\alpha e_b{}^i e^c{}_i, \\
 K_{\alpha\beta a} &= 2D_{[\alpha}A_{a\beta]} + A_{m\alpha}A_{n\beta}\tilde{f}_a{}^{mn} + 2A_{i[\alpha}\partial_{\beta]}e_a{}^i,
 \end{aligned} \tag{C.6}$$

while for $A_a{}^\alpha = B = 0$ they can be expanded as

$$\begin{aligned}
 J^c{}_{ab} &= -\partial_b \tilde{A}^c{}_\alpha + \tilde{A}^m{}_\alpha f^c{}_{mb} + \partial_\alpha e_b{}^i e^c{}_i, \\
 \tilde{G}_\alpha{}^{ab} &= -D_\alpha\beta^{ab} + D_\alpha\tilde{A}^{[a\gamma}\tilde{A}^{b]}\gamma - 2\tilde{D}^{[a}\tilde{A}_\alpha{}^{b]} + \tilde{A}^m{}_\alpha\tilde{f}_m{}^{ab} + 2\tilde{C}^{[am}\tilde{A}^n{}_\alpha\tilde{f}_{n]}^{b]} \\
 &\quad + 2(\tilde{C}^{[ai}\partial_\alpha e_{b]}^i - \tilde{A}^i{}_\alpha\tilde{A}^{[a\gamma}\partial_\gamma e_{b]}^i), \\
 \tilde{K}^{\alpha\beta a} &= 2D^{[\alpha}\tilde{A}^{a\beta]} + \tilde{A}^{m\alpha}\tilde{A}^{n\beta}f^a{}_{mn} + 2\tilde{A}^{i[\alpha}\partial^{\beta]}e^a{}_i.
 \end{aligned} \tag{C.7}$$

In addition, there exists the flux

$$F_\alpha = -\partial_m \tilde{A}^m{}_\alpha - \tilde{\partial}^m A_{m\alpha} + 2D_\alpha d. \tag{C.8}$$

Bibliography

- [1] **CMS** Collaboration, S. Chatrchyan *et al.*, “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” *Phys. Lett. B* **716** (2012) 30–61, [arXiv:1207.7235 \[hep-ex\]](#).
- [2] **ATLAS** Collaboration, G. Aad *et al.*, “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” *Phys. Lett. B* **716** (2012) 1–29, [arXiv:1207.7214 \[hep-ex\]](#).
- [3] **ATLAS** Collaboration, “Combined coupling measurements of the Higgs-like boson with the ATLAS detector using up to 25 fb^{−1} of proton-proton collision data,”.
- [4] **CMS** Collaboration, C. Collaboration, “Search for the standard model Higgs boson produced in association with W or Z bosons, and decaying to bottom quarks for LHCp 2013,”.
- [5] **BICEP2** Collaboration, P. A. R. Ade *et al.*, “Detection of *B*-Mode Polarization at Degree Angular Scales by BICEP2,” *Phys. Rev. Lett.* **112** no. 24, (2014) 241101, [arXiv:1403.3985 \[astro-ph.CO\]](#).
- [6] **Planck** Collaboration, P. A. R. Ade *et al.*, “Planck 2015 results. XX. Constraints on inflation,” [arXiv:1502.02114 \[astro-ph.CO\]](#).
- [7] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall, “Extra natural inflation,” *Phys. Rev. Lett.* **90** (2003) 221302, [arXiv:hep-th/0301218 \[hep-th\]](#).
- [8] T. Rudelius, “On the Possibility of Large Axion Moduli Spaces,” *JCAP* **1504** no. 04, (2015) 049, [arXiv:1409.5793 \[hep-th\]](#).
- [9] T. C. Bachlechner, C. Long, and L. McAllister, “Planckian Axions in String Theory,” *JHEP* **12** (2015) 042, [arXiv:1412.1093 \[hep-th\]](#).
- [10] A. de la Fuente, P. Saraswat, and R. Sundrum, “Natural Inflation and Quantum Gravity,” *Phys. Rev. Lett.* **114** no. 15, (2015) 151303, [arXiv:1412.3457 \[hep-th\]](#).
- [11] T. Rudelius, “Constraints on Axion Inflation from the Weak Gravity Conjecture,” *JCAP* **1509** no. 09, (2015) 020, [arXiv:1503.00795 \[hep-th\]](#).
- [12] M. Montero, A. M. Uranga, and I. Valenzuela, “Transplanckian axions!?,” *JHEP* **08** (2015) 032, [arXiv:1503.03886 \[hep-th\]](#).
- [13] J. Brown, W. Cottrell, G. Shiu, and P. Soler, “Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation,” *JHEP* **10** (2015) 023, [arXiv:1503.04783 \[hep-th\]](#).

- [14] T. C. Bachlechner, C. Long, and L. McAllister, “Planckian Axions and the Weak Gravity Conjecture,” *JHEP* **01** (2016) 091, [arXiv:1503.07853 \[hep-th\]](#).
- [15] A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski, “Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?,” *Phys. Lett.* **B748** (2015) 455–462, [arXiv:1503.07912 \[hep-th\]](#).
- [16] J. Brown, W. Cottrell, G. Shiu, and P. Soler, “On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture,” *JHEP* **04** (2016) 017, [arXiv:1504.00659 \[hep-th\]](#).
- [17] D. Junghans, “Large-Field Inflation with Multiple Axions and the Weak Gravity Conjecture,” *JHEP* **02** (2016) 128, [arXiv:1504.03566 \[hep-th\]](#).
- [18] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, “Systematics of moduli stabilisation in Calabi-Yau flux compactifications,” *JHEP* **0503** (2005) 007, [arXiv:hep-th/0502058 \[hep-th\]](#).
- [19] J. P. Conlon, S. S. Abdussalam, F. Quevedo, and K. Suruliz, “Soft SUSY Breaking Terms for Chiral Matter in IIB String Compactifications,” *JHEP* **01** (2007) 032, [arXiv:hep-th/0610129 \[hep-th\]](#).
- [20] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys.Rev.* **D68** (2003) 046005, [arXiv:hep-th/0301240 \[hep-th\]](#).
- [21] B. Vercnocke and T. Wrase, “Constrained superfields from an anti-D3-brane in KKLT,” *JHEP* **08** (2016) 132, [arXiv:1605.03961 \[hep-th\]](#).
- [22] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen, and T. Wrase, “ $\overline{D3}$ and dS,” *JHEP* **05** (2015) 058, [arXiv:1502.07627 \[hep-th\]](#).
- [23] N. Kaloper and L. Sorbo, “A Natural Framework for Chaotic Inflation,” *Phys.Rev.Lett.* **102** (2009) 121301, [arXiv:0811.1989 \[hep-th\]](#).
- [24] N. Kaloper, A. Lawrence, and L. Sorbo, “An Ignoble Approach to Large Field Inflation,” *JCAP* **1103** (2011) 023, [arXiv:1101.0026 \[hep-th\]](#).
- [25] E. Silverstein and A. Westphal, “Monodromy in the CMB: Gravity Waves and String Inflation,” *Phys.Rev.* **D78** (2008) 106003, [arXiv:0803.3085 \[hep-th\]](#).
- [26] L. McAllister, E. Silverstein, and A. Westphal, “Gravity Waves and Linear Inflation from Axion Monodromy,” *Phys.Rev.* **D82** (2010) 046003, [arXiv:0808.0706 \[hep-th\]](#).
- [27] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge University Press, 2015. [arXiv:1404.2601 \[hep-th\]](#).

-
- [28] A. Westphal, “String cosmology — Large-field inflation in string theory,” *Int. J. Mod. Phys. A* **30** no. 09, (2015) 1530024, [arXiv:1409.5350 \[hep-th\]](#).
- [29] M. Arends, A. Hebecker, K. Heimpel, S. C. Kraus, D. Lüster, C. Mayrhofer, C. Schick, and T. Weigand, “D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation,” *Fortsch. Phys.* **62** (2014) 647–702, [arXiv:1405.0283 \[hep-th\]](#).
- [30] L. McAllister, E. Silverstein, A. Westphal, and T. Wrase, “The Powers of Monodromy,” *JHEP* **09** (2014) 123, [arXiv:1405.3652 \[hep-th\]](#).
- [31] S. Franco, D. Galloni, A. Retolaza, and A. Uranga, “On axion monodromy inflation in warped throats,” *JHEP* **02** (2015) 086, [arXiv:1405.7044 \[hep-th\]](#).
- [32] F. Hassler, D. Lüster, and S. Massai, “On Inflation and de Sitter in Non-Geometric String Backgrounds,” [arXiv:1405.2325 \[hep-th\]](#).
- [33] Z. Kenton and S. Thomas, “D-brane Potentials in the Warped Resolved Conifold and Natural Inflation,” *JHEP* **02** (2015) 127, [arXiv:1409.1221 \[hep-th\]](#).
- [34] K. Kooner, S. Parameswaran, and I. Zavala, “Warping the Weak Gravity Conjecture,” [arXiv:1509.07049 \[hep-th\]](#).
- [35] W. Buchmüller, E. Dudas, L. Heurtier, A. Westphal, C. Wieck, and M. W. Winkler, “Challenges for Large-Field Inflation and Moduli Stabilization,” *JHEP* **04** (2015) 058, [arXiv:1501.05812 \[hep-th\]](#).
- [36] L. E. Ibáñez and I. Valenzuela, “The inflaton as an MSSM Higgs and open string modulus monodromy inflation,” *Phys. Lett. B* **736** (2014) 226–230, [arXiv:1404.5235 \[hep-th\]](#).
- [37] T. W. Grimm, “Axion Inflation in F-theory,” *Phys. Lett. B* **739** (2014) 201–208, [arXiv:1404.4268 \[hep-th\]](#).
- [38] I. García-Etxebarria, T. W. Grimm, and I. Valenzuela, “Special Points of Inflation in Flux Compactifications,” *Nucl. Phys. B* **899** (2015) 414–443, [arXiv:1412.5537 \[hep-th\]](#).
- [39] A. Achúcarro, B. de Carlos, J. A. Casas, and L. Doplicher, “De Sitter vacua from uplifting D-terms in effective supergravities from realistic strings,” *JHEP* **06** (2006) 014, [arXiv:hep-th/0601190 \[hep-th\]](#).
- [40] J. Gray, Y.-H. He, A. Ilderton, and A. Lukas, “STRINGVACUA: A Mathematica Package for Studying Vacuum Configurations in String Phenomenology,” *Comput. Phys. Commun.* **180** (2009) 107–119, [arXiv:0801.1508 \[hep-th\]](#).

- [41] S. Krippendorff and F. Quevedo, “Metastable SUSY Breaking, de Sitter Moduli Stabilisation and Kähler Moduli Inflation,” *JHEP* **11** (2009) 039, [arXiv:0901.0683 \[hep-th\]](#).
- [42] U. Danielsson and G. Dibitetto, “On the distribution of stable de Sitter vacua,” *JHEP* **03** (2013) 018, [arXiv:1212.4984 \[hep-th\]](#).
- [43] J. Blåbäck, U. Danielsson, and G. Dibitetto, “Fully stable dS vacua from generalised fluxes,” *JHEP* **08** (2013) 054, [arXiv:1301.7073 \[hep-th\]](#).
- [44] C. Damian, L. R. Diaz-Barron, O. Loaiza-Brito, and M. Sabido, “Slow-Roll Inflation in Non-geometric Flux Compactification,” *JHEP* **06** (2013) 109, [arXiv:1302.0529 \[hep-th\]](#).
- [45] C. Damian and O. Loaiza-Brito, “More stable de Sitter vacua from S-dual nongeometric fluxes,” *Phys. Rev.* **D88** no. 4, (2013) 046008, [arXiv:1304.0792 \[hep-th\]](#).
- [46] R. Kallosh, A. Linde, B. Vercnocke, and T. Wrase, “Analytic Classes of Metastable de Sitter Vacua,” *JHEP* **10** (2014) 11, [arXiv:1406.4866 \[hep-th\]](#).
- [47] M. Rummel and Y. Sumitomo, “De Sitter Vacua from a D-term Generated Racetrack Uplift,” *JHEP* **01** (2015) 015, [arXiv:1407.7580 \[hep-th\]](#).
- [48] J. Blåbäck, U. H. Danielsson, G. Dibitetto, and S. C. Vargas, “Universal dS vacua in STU-models,” [arXiv:1505.04283 \[hep-th\]](#).
- [49] M. P. Hertzberg, S. Kachru, W. Taylor, and M. Tegmark, “Inflationary Constraints on Type IIA String Theory,” *JHEP* **12** (2007) 095, [arXiv:0711.2512 \[hep-th\]](#).
- [50] S. S. Haque, G. Shiu, B. Underwood, and T. Van Riet, “Minimal simple de Sitter solutions,” *Phys. Rev.* **D79** (2009) 086005, [arXiv:0810.5328 \[hep-th\]](#).
- [51] R. Flauger, S. Paban, D. Robbins, and T. Wrase, “Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes,” *Phys. Rev.* **D79** (2009) 086011, [arXiv:0812.3886 \[hep-th\]](#).
- [52] U. H. Danielsson, S. S. Haque, G. Shiu, and T. Van Riet, “Towards Classical de Sitter Solutions in String Theory,” *JHEP* **09** (2009) 114, [arXiv:0907.2041 \[hep-th\]](#).
- [53] B. de Carlos, A. Guarino, and J. M. Moreno, “Flux moduli stabilisation, Supergravity algebras and no-go theorems,” *JHEP* **1001** (2010) 012, [arXiv:0907.5580 \[hep-th\]](#).
- [54] C. Caviezel, T. Wrase, and M. Zagermann, “Moduli Stabilization and Cosmology of Type IIB on SU(2)-Structure Orientifolds,” *JHEP* **04** (2010) 011, [arXiv:0912.3287 \[hep-th\]](#).

-
- [55] T. Wrase and M. Zagermann, “On Classical de Sitter Vacua in String Theory,” *Fortsch. Phys.* **58** (2010) 906–910, [arXiv:1003.0029 \[hep-th\]](#).
- [56] G. Shiu and Y. Sumitomo, “Stability Constraints on Classical de Sitter Vacua,” *JHEP* **09** (2011) 052, [arXiv:1107.2925 \[hep-th\]](#).
- [57] A. Borghese, D. Roest, and I. Zavala, “A Geometric bound on F-term inflation,” *JHEP* **09** (2012) 021, [arXiv:1203.2909 \[hep-th\]](#).
- [58] S. R. Green, E. J. Martinec, C. Quigley, and S. Sethi, “Constraints on String Cosmology,” *Class. Quant. Grav.* **29** (2012) 075006, [arXiv:1110.0545 \[hep-th\]](#).
- [59] F. F. Gautason, D. Junghans, and M. Zagermann, “On Cosmological Constants from α' -Corrections,” *JHEP* **06** (2012) 029, [arXiv:1204.0807 \[hep-th\]](#).
- [60] D. Kutasov, T. Maxfield, I. Melnikov, and S. Sethi, “Constraining de Sitter Space in String Theory,” *Phys. Rev. Lett.* **115** no. 7, (2015) 071305, [arXiv:1504.00056 \[hep-th\]](#).
- [61] J. Shelton, W. Taylor, and B. Wecht, “Nongeometric flux compactifications,” *JHEP* **0510** (2005) 085, [arXiv:hep-th/0508133 \[hep-th\]](#).
- [62] J. Shelton, W. Taylor, and B. Wecht, “Generalized Flux Vacua,” *JHEP* **0702** (2007) 095, [arXiv:hep-th/0607015 \[hep-th\]](#).
- [63] G. Aldazabal, D. Marqués, and C. Núñez, “Double Field Theory: A Pedagogical Review,” *Class. Quant. Grav.* **30** (2013) 163001, [arXiv:1305.1907 \[hep-th\]](#).
- [64] D. S. Berman and D. C. Thompson, “Duality Symmetric String and M-Theory,” [arXiv:1306.2643 \[hep-th\]](#).
- [65] D. Andriot, M. Larfors, D. Lüst, and P. Patalong, “A ten-dimensional action for non-geometric fluxes,” *JHEP* **09** (2011) 134, [arXiv:1106.4015 \[hep-th\]](#).
- [66] D. Geissbühler, D. Marqués, C. Núñez, and V. Penas, “Exploring Double Field Theory,” *JHEP* **1306** (2013) 101, [arXiv:1304.1472 \[hep-th\]](#).
- [67] R. Blumenhagen, X. Gao, D. Herschmann, and P. Shukla, “Dimensional Oxidation of Non-geometric Fluxes in Type II Orientifolds,” *JHEP* **1310** (2013) 201, [arXiv:1306.2761 \[hep-th\]](#).
- [68] A. Font, A. Guarino, and J. M. Moreno, “Algebras and non-geometric flux vacua,” *JHEP* **12** (2008) 050, [arXiv:0809.3748 \[hep-th\]](#).
- [69] A. Guarino and G. J. Weatherill, “Non-geometric flux vacua, S-duality and algebraic geometry,” *JHEP* **0902** (2009) 042, [arXiv:0811.2190 \[hep-th\]](#).

- [70] B. de Carlos, A. Guarino, and J. M. Moreno, “Complete classification of Minkowski vacua in generalised flux models,” *JHEP* **02** (2010) 076, [arXiv:0911.2876 \[hep-th\]](#).
- [71] G. Aldazabal, D. Marqués, C. Núñez, and J. A. Rosabal, “On Type IIB moduli stabilization and $N = 4, 8$ supergravities,” *Nucl.Phys.* **B849** (2011) 80–111, [arXiv:1101.5954 \[hep-th\]](#).
- [72] G. Dibitetto, A. Guarino, and D. Roest, “Charting the landscape of $N=4$ flux compactifications,” *JHEP* **1103** (2011) 137, [arXiv:1102.0239 \[hep-th\]](#).
- [73] A. Dabholkar and C. Hull, “Duality twists, orbifolds, and fluxes,” *JHEP* **0309** (2003) 054, [arXiv:hep-th/0210209 \[hep-th\]](#).
- [74] C. Hull, “A Geometry for non-geometric string backgrounds,” *JHEP* **0510** (2005) 065, [arXiv:hep-th/0406102 \[hep-th\]](#).
- [75] A. Dabholkar and C. Hull, “Generalised T-duality and non-geometric backgrounds,” *JHEP* **0605** (2006) 009, [arXiv:hep-th/0512005 \[hep-th\]](#).
- [76] T. Buscher, “A Symmetry of the String Background Field Equations,” *Phys.Lett.* **B194** (1987) 59.
- [77] T. Buscher, “Path Integral Derivation of Quantum Duality in Nonlinear Sigma Models,” *Phys.Lett.* **B201** (1988) 466.
- [78] W. Siegel, “Two vierbein formalism for string inspired axionic gravity,” *Phys.Rev.* **D47** (1993) 5453–5459, [arXiv:hep-th/9302036 \[hep-th\]](#).
- [79] W. Siegel, “Superspace duality in low-energy superstrings,” *Phys.Rev.* **D48** (1993) 2826–2837, [arXiv:hep-th/9305073 \[hep-th\]](#).
- [80] O. Hohm and S. K. Kwak, “Frame-like Geometry of Double Field Theory,” *J.Phys.* **A44** (2011) 085404, [arXiv:1011.4101 \[hep-th\]](#).
- [81] C. Hull and B. Zwiebach, “Double Field Theory,” *JHEP* **0909** (2009) 099, [arXiv:0904.4664 \[hep-th\]](#).
- [82] O. Hohm, C. Hull, and B. Zwiebach, “Background independent action for double field theory,” *JHEP* **07** (2010) 016, [arXiv:1003.5027 \[hep-th\]](#).
- [83] O. Hohm, C. Hull, and B. Zwiebach, “Generalized metric formulation of double field theory,” *JHEP* **08** (2010) 008, [arXiv:1006.4823 \[hep-th\]](#).
- [84] O. Hohm, D. Lüst, and B. Zwiebach, “The Spacetime of Double Field Theory: Review, Remarks, and Outlook,” *Fortsch.Phys.* **61** (2013) 926–966, [arXiv:1309.2977 \[hep-th\]](#).

-
- [85] N. Hitchin, “Generalized Calabi-Yau manifolds,” *Quart.J.Math.Oxford Ser.* **54** (2003) 281–308, [arXiv:math/0209099](#) [math-dg].
 - [86] M. Gualtieri, “Generalized complex geometry,” [arXiv:math/0401221](#) [math-dg].
 - [87] M. Graña, R. Minasian, M. Petrini, and D. Waldram, “T-duality, Generalized Geometry and Non-Geometric Backgrounds,” *JHEP* **0904** (2009) 075, [arXiv:0807.4527](#) [hep-th].
 - [88] A. Coimbra, C. Strickland-Constable, and D. Waldram, “Supergravity as Generalised Geometry I: Type II Theories,” *JHEP* **1111** (2011) 091, [arXiv:1107.1733](#) [hep-th].
 - [89] D. Andriot, O. Hohm, M. Larfors, D. Lüst, and P. Patalong, “Non-Geometric Fluxes in Supergravity and Double Field Theory,” *Fortsch.Phys.* **60** (2012) 1150–1186, [arXiv:1204.1979](#) [hep-th].
 - [90] D. Andriot and A. Betz, “ β -supergravity: a ten-dimensional theory with non-geometric fluxes, and its geometric framework,” *JHEP* **1312** (2013) 083, [arXiv:1306.4381](#) [hep-th].
 - [91] R. Blumenhagen, A. Deser, E. Plauschinn, and F. Rennecke, “Non-geometric strings, symplectic gravity and differential geometry of Lie algebroids,” *JHEP* **1302** (2013) 122, [arXiv:1211.0030](#) [hep-th].
 - [92] R. Blumenhagen, A. Deser, E. Plauschinn, F. Rennecke, and C. Schmid, “The Intriguing Structure of Non-geometric Frames in String Theory,” *Fortsch.Phys.* **61** (2013) 893–925, [arXiv:1304.2784](#) [hep-th].
 - [93] R. Reid-Edwards and B. Spanjaard, “N=4 Gauged Supergravity from Duality-Twist Compactifications of String Theory,” *JHEP* **0812** (2008) 052, [arXiv:0810.4699](#) [hep-th].
 - [94] O. Hohm and S. K. Kwak, “Double Field Theory Formulation of Heterotic Strings,” *JHEP* **1106** (2011) 096, [arXiv:1103.2136](#) [hep-th].
 - [95] D. Andriot, “Heterotic string from a higher dimensional perspective,” *Nucl.Phys.* **B855** (2012) 222–267, [arXiv:1102.1434](#) [hep-th].
 - [96] M. Garcia-Fernandez, “Torsion-free generalized connections and Heterotic Supergravity,” *Commun.Math.Phys.* **332** no. 1, (2014) 89–115, [arXiv:1304.4294](#) [math.DG].
 - [97] D. Baraglia and P. Hekmati, “Transitive Courant Algebroids, String Structures and T-duality,” [arXiv:1308.5159](#) [math.DG].

- [98] L. B. Anderson, J. Gray, and E. Sharpe, “Algebroids, Heterotic Moduli Spaces and the Strominger System,” *JHEP* **1407** (2014) 037, [arXiv:1402.1532 \[hep-th\]](#).
- [99] X. de la Ossa and E. E. Svanes, “Holomorphic Bundles and the Moduli Space of N=1 Supersymmetric Heterotic Compactifications,” *JHEP* **1410** (2014) 123, [arXiv:1402.1725 \[hep-th\]](#).
- [100] M. Graña and D. Marqués, “Gauged Double Field Theory,” *JHEP* **1204** (2012) 020, [arXiv:1201.2924 \[hep-th\]](#).
- [101] O. A. Bedoya, D. Marqués, and C. Núñez, “Heterotic α' -corrections in Double Field Theory,” [arXiv:1407.0365 \[hep-th\]](#).
- [102] R. Blumenhagen and R. Sun, “T-duality, Non-geometry and Lie Algebroids in Heterotic Double Field Theory,” *JHEP* **02** (2015) 097, [arXiv:1411.3167 \[hep-th\]](#).
- [103] A. Coimbra, R. Minasian, H. Triendl, and D. Waldram, “Generalised geometry for string corrections,” [arXiv:1407.7542 \[hep-th\]](#).
- [104] X. de la Ossa and E. E. Svanes, “Connections, Field Redefinitions and Heterotic Supergravity,” [arXiv:1409.3347 \[hep-th\]](#).
- [105] O. Hohm, W. Siegel, and B. Zwiebach, “Doubled α' -geometry,” *JHEP* **1402** (2014) 065, [arXiv:1306.2970 \[hep-th\]](#).
- [106] O. Hohm and B. Zwiebach, “Green-Schwarz mechanism and α' -deformed Courant brackets,” [arXiv:1407.0708 \[hep-th\]](#).
- [107] O. Hohm and B. Zwiebach, “Double Field Theory at Order α' ,” [arXiv:1407.3803 \[hep-th\]](#).
- [108] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” *JHEP* **9909** (1999) 032, [arXiv:hep-th/9908142 \[hep-th\]](#).
- [109] M. Herbst, A. Kling, and M. Kreuzer, “Star products from open strings in curved backgrounds,” *JHEP* **0109** (2001) 014, [arXiv:hep-th/0106159 \[hep-th\]](#).
- [110] L. Cornalba and R. Schiappa, “Nonassociative star product deformations for D-brane world volumes in curved backgrounds,” *Commun.Math.Phys.* **225** (2002) 33–66, [arXiv:hep-th/0101219 \[hep-th\]](#).
- [111] M. Herbst, A. Kling, and M. Kreuzer, “Noncommutative tachyon action and D-brane geometry,” *JHEP* **08** (2002) 010, [arXiv:hep-th/0203077 \[hep-th\]](#).
- [112] R. Blumenhagen and E. Plauschinn, “Nonassociative Gravity in String Theory?,” *J.Phys.* **A44** (2011) 015401, [arXiv:1010.1263 \[hep-th\]](#).

-
- [113] R. Blumenhagen, A. Deser, D. Lüst, E. Plauschinn, and F. Rennecke, “Non-geometric Fluxes, Asymmetric Strings and Nonassociative Geometry,” *J.Phys.* **A44** (2011) 385401, [arXiv:1106.0316 \[hep-th\]](#).
- [114] D. Robbins and T. Wrase, “D-terms from generalized NS-NS fluxes in type II,” *JHEP* **12** (2007) 058, [arXiv:0709.2186 \[hep-th\]](#).
- [115] R. Blumenhagen, D. Lüst, and S. Theisen, *Basic concepts of string theory*. Springer, Heidelberg, Germany, 2013.
- [116] B. Zwiebach, *A first course in string theory*. Cambridge University Press, 2006.
- [117] L. E. Ibanez and A. M. Uranga, *String theory and particle physics: An introduction to string phenomenology*. Cambridge University Press, 2012.
- [118] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory. Vol. 1: Introduction*. Cambridge University Press, Cambridge, UK, 1988.
- [119] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory. Vol. 2: Loop amplitudes, anomalies and phenomenology*. Cambridge University Press, Cambridge, UK, 1988.
- [120] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge University Press, Cambridge, UK, 2007.
- [121] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*. Cambridge University Press, Cambridge, UK, 2007.
- [122] G. Aldazabal, W. Baron, D. Marqués, and C. Núñez, “The effective action of Double Field Theory,” *JHEP* **1111** (2011) 052, [arXiv:1109.0290 \[hep-th\]](#).
- [123] D. Geissbuhler, “Double Field Theory and N=4 Gauged Supergravity,” *JHEP* **1111** (2011) 116, [arXiv:1109.4280 \[hep-th\]](#).
- [124] D. S. Berman, C. D. A. Blair, E. Malek, and M. J. Perry, “The O(D,D) Geometry of String Theory,” [arXiv:1303.6727 \[hep-th\]](#).
- [125] E. Cremmer, B. Julia, and J. Scherk, “Supergravity Theory in Eleven-Dimensions,” *Phys. Lett.* **B76** (1978) 409–412.
- [126] B. Julia, “Group Disintegrations in Superspace and Supergravity,” *Proceedings of the Nuffield Workshop Cambridge* **1980** (1981) .
- [127] C. D. A. Blair and E. Malek, “Geometry and fluxes of SL(5) exceptional field theory,” *JHEP* **03** (2015) 144, [arXiv:1412.0635 \[hep-th\]](#).
- [128] M. Graña, J. Louis, and D. Waldram, “Hitchin functionals in N=2 supergravity,” *JHEP* **0601** (2006) 008, [arXiv:hep-th/0505264 \[hep-th\]](#).

- [129] I. Benmachiche and T. W. Grimm, “Generalized N=1 orientifold compactifications and the Hitchin functionals,” *Nucl.Phys.* **B748** (2006) 200–252, [arXiv:hep-th/0602241](#) [[hep-th](#)].
- [130] M. Graña, J. Louis, and D. Waldram, “SU(3) x SU(3) compactification and mirror duals of magnetic fluxes,” *JHEP* **0704** (2007) 101, [arXiv:hep-th/0612237](#) [[hep-th](#)].
- [131] A. Micu, E. Palti, and G. Tasinato, “Towards Minkowski Vacua in Type II String Compactifications,” *JHEP* **0703** (2007) 104, [arXiv:hep-th/0701173](#) [[hep-th](#)].
- [132] E. Palti, “Low Energy Supersymmetry from Non-Geometry,” *JHEP* **0710** (2007) 011, [arXiv:0707.1595](#) [[hep-th](#)].
- [133] T. W. Grimm and J. Louis, “The Effective action of N = 1 Calabi-Yau orientifolds,” *Nucl.Phys.* **B699** (2004) 387–426, [arXiv:hep-th/0403067](#) [[hep-th](#)].
- [134] R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, E. Plauschinn, Y. Sekiguchi, and F. Wolf, “A Flux-Scaling Scenario for High-Scale Moduli Stabilization in String Theory,” *Nucl. Phys.* **B897** (2015) 500–554, [arXiv:1503.07634](#) [[hep-th](#)].
- [135] R. Blumenhagen, A. Font, and E. Plauschinn, “Relating Double Field Theory to the Scalar Potential of N=2 Gauged Supergravity,” [arXiv:1507.08059](#) [[hep-th](#)].
- [136] J. Wess and J. Bagger, *Supersymmetry and supergravity*. Princeton University Press, 1992.
- [137] H. Jockers, “The Effective action of D-branes in Calabi-Yau orientifold compactifications,” *Fortsch. Phys.* **53** (2005) 1087–1175, [arXiv:hep-th/0507042](#) [[hep-th](#)].
- [138] S. J. Gates, M. T. Grisaru, M. Rocek, and W. Siegel, “Superspace Or One Thousand and One Lessons in Supersymmetry,” *Front. Phys.* **58** (1983) 1–548, [arXiv:hep-th/0108200](#) [[hep-th](#)].
- [139] R. D’Auria, S. Ferrara, and M. Trigiante, “On the supergravity formulation of mirror symmetry in generalized Calabi-Yau manifolds,” *Nucl.Phys.* **B780** (2007) 28–39, [arXiv:hep-th/0701247](#) [[HEP-TH](#)].
- [140] G. Aldazabal, P. G. Camara, A. Font, and L. Ibáñez, “More dual fluxes and moduli fixing,” *JHEP* **0605** (2006) 070, [arXiv:hep-th/0602089](#) [[hep-th](#)].
- [141] R. Blumenhagen, “A Course on Noncommutative Geometry in String Theory,” *Fortsch.Phys.* **62** (2014) 709–726, [arXiv:1403.4805](#) [[hep-th](#)].
- [142] E. Bergshoeff, I. Entrop, and R. Kallosh, “Exact duality in string effective action,” *Phys.Rev.* **D49** (1994) 6663–6673, [arXiv:hep-th/9401025](#) [[hep-th](#)].

-
- [143] M. Serone and M. Trapletti, “A Note on T-duality in heterotic string theory,” *Phys.Lett.* **B637** (2006) 331–337, [arXiv:hep-th/0512272](#) [[hep-th](#)].
- [144] J. Polchinski and E. Witten, “Evidence for heterotic - type I string duality,” *Nucl.Phys.* **B460** (1996) 525–540, [arXiv:hep-th/9510169](#) [[hep-th](#)].
- [145] C. M. Hull, “Generalised Geometry for M-Theory,” *JHEP* **07** (2007) 079, [arXiv:hep-th/0701203](#) [[hep-th](#)].
- [146] A. Coimbra, C. Strickland-Constable, and D. Waldram, “ $E_{d(d)} \times \mathbb{R}^+$ generalised geometry, connections and M theory,” *JHEP* **02** (2014) 054, [arXiv:1112.3989](#) [[hep-th](#)].
- [147] B. Wecht, “Lectures on Nongeometric Flux Compactifications,” *Class.Quant.Grav.* **24** (2007) S773–S794, [arXiv:0708.3984](#) [[hep-th](#)].
- [148] D. S. Freed and E. Witten, “Anomalies in string theory with D-branes,” *Asian J.Math* **3** (1999) 819, [arXiv:hep-th/9907189](#) [[hep-th](#)].
- [149] P. Bouwknegt, K. Hannabuss, and V. Mathai, “Nonassociative tori and applications to T-duality,” *Commun.Math.Phys.* **264** (2006) 41–69, [arXiv:hep-th/0412092](#) [[hep-th](#)].
- [150] D. Lüster, “T-duality and closed string non-commutative (doubled) geometry,” *JHEP* **1012** (2010) 084, [arXiv:1010.1361](#) [[hep-th](#)].
- [151] C. Condeescu, I. Florakis, and D. Lüster, “Asymmetric Orbifolds, Non-Geometric Fluxes and Non-Commutativity in Closed String Theory,” *JHEP* **1204** (2012) 121, [arXiv:1202.6366](#) [[hep-th](#)].
- [152] D. Andriot, M. Larfors, D. Lust, and P. Patalong, “(Non-)commutative closed string on T-dual toroidal backgrounds,” *JHEP* **1306** (2013) 021, [arXiv:1211.6437](#) [[hep-th](#)].
- [153] A. Chatzistavrakidis and L. Jonke, “Matrix theory origins of non-geometric fluxes,” *JHEP* **1302** (2013) 040, [arXiv:1207.6412](#) [[hep-th](#)].
- [154] D. Mylonas, P. Schupp, and R. J. Szabo, “Membrane Sigma-Models and Quantization of Non-Geometric Flux Backgrounds,” *JHEP* **1209** (2012) 012, [arXiv:1207.0926](#) [[hep-th](#)].
- [155] I. Bakas and D. Lüster, “3-Cocycles, Non-Associative Star-Products and the Magnetic Paradigm of R-Flux String Vacua,” [arXiv:1309.3172](#) [[hep-th](#)].
- [156] C. Hull and B. Zwiebach, “The Gauge algebra of double field theory and Courant brackets,” *JHEP* **0909** (2009) 090, [arXiv:0908.1792](#) [[hep-th](#)].

- [157] B. Zwiebach, “Double Field Theory, T-Duality, and Courant Brackets,” *Lect.Notes Phys.* **851** (2012) 265–291, [arXiv:1109.1782 \[hep-th\]](#).
- [158] G. Dibitetto, J. Fernandez-Melgarejo, D. Marques, and D. Roest, “Duality orbits of non-geometric fluxes,” *Fortsch.Phys.* **60** (2012) 1123–1149, [arXiv:1203.6562 \[hep-th\]](#).
- [159] O. Hohm and S. K. Kwak, “Massive Type II in Double Field Theory,” *JHEP* **1111** (2011) 086, [arXiv:1108.4937 \[hep-th\]](#).
- [160] J. Scherk and J. H. Schwarz, “Spontaneous Breaking of Supersymmetry Through Dimensional Reduction,” *Phys.Lett.* **B82** (1979) 60.
- [161] J. Scherk and J. H. Schwarz, “How to Get Masses from Extra Dimensions,” *Nucl.Phys.* **B153** (1979) 61–88.
- [162] D. S. Berman, E. T. Musaev, D. C. Thompson, and D. C. Thompson, “Duality Invariant M-theory: Gauged supergravities and Scherk-Schwarz reductions,” *JHEP* **1210** (2012) 174, [arXiv:1208.0020 \[hep-th\]](#).
- [163] D. S. Berman and K. Lee, “Supersymmetry for Gauged Double Field Theory and Generalised Scherk-Schwarz Reductions,” [arXiv:1305.2747 \[hep-th\]](#).
- [164] C. Condeescu, I. Florakis, C. Kounnas, and D. Lüst, “Gauged supergravities and non-geometric Q/R-fluxes from asymmetric orbifold CFT’s,” *JHEP* **1310** (2013) 057, [arXiv:1307.0999 \[hep-th\]](#).
- [165] O. Hohm and B. Zwiebach, “Large Gauge Transformations in Double Field Theory,” *JHEP* **1302** (2013) 075, [arXiv:1207.4198 \[hep-th\]](#).
- [166] M. Herbst, A. Kling, and M. Kreuzer, “Cyclicity of nonassociative products on D-branes,” *JHEP* **0403** (2004) 003, [arXiv:hep-th/0312043 \[hep-th\]](#).
- [167] R. Jackiw, “3 - Cocycle in Mathematics and Physics,” *Phys.Rev.Lett.* **54** (1985) 159–162.
- [168] R. Jackiw, “Magnetic sources and three cocycles (comment),” *Phys.Lett.* **B154** (1985) 303–304.
- [169] Y.-S. Wu and A. Zee, “Cocycles and Magnetic Monopoles,” *Phys.Lett.* **B152** (1985) 98.
- [170] B. Grossman, “A Three Cocycle in Quantum Mechanics,” *Phys.Lett.* **B152** (1985) 93.
- [171] B. Grossman, “The 3 Cocycle in Quantum Mechanics. 2,” *Phys.Rev.* **D33** (1986) 2922.

-
- [172] M. Kontsevich, “Deformation quantization of Poisson manifolds. 1.,” *Lett.Math.Phys.* **66** (2003) 157–216, [arXiv:q-alg/9709040 \[q-alg\]](#).
- [173] O. Hohm, S. K. Kwak, and B. Zwiebach, “Unification of Type II Strings and T-duality,” *Phys.Rev.Lett.* **107** (2011) 171603, [arXiv:1106.5452 \[hep-th\]](#).
- [174] O. Hohm, S. K. Kwak, and B. Zwiebach, “Double Field Theory of Type II Strings,” *JHEP* **1109** (2011) 013, [arXiv:1107.0008 \[hep-th\]](#).
- [175] F. Marchesano, G. Shiu, and A. M. Uranga, “F-term Axion Monodromy Inflation,” *JHEP* **09** (2014) 184, [arXiv:1404.3040 \[hep-th\]](#).
- [176] A. Hebecker, S. C. Kraus, and L. T. Witkowski, “D7-Brane Chaotic Inflation,” *Phys. Lett.* **B737** (2014) 16–22, [arXiv:1404.3711 \[hep-th\]](#).
- [177] R. Blumenhagen and E. Plauschinn, “Towards Universal Axion Inflation and Reheating in String Theory,” *Phys. Lett.* **B736** (2014) 482–487, [arXiv:1404.3542 \[hep-th\]](#).
- [178] R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, and E. Plauschinn, “Towards Axionic Starobinsky-like Inflation in String Theory,” *Phys. Lett.* **B746** (2015) 217–222, [arXiv:1503.01607 \[hep-th\]](#).
- [179] R. Kallosh and T. Wrase, “Emergence of Spontaneously Broken Supersymmetry on an Anti-D3-Brane in KKLT dS Vacua,” *JHEP* **12** (2014) 117, [arXiv:1411.1121 \[hep-th\]](#).
- [180] R. Kallosh, F. Quevedo, and A. M. Uranga, “String Theory Realizations of the Nilpotent Goldstino,” [arXiv:1507.07556 \[hep-th\]](#).
- [181] J. Polchinski, “Brane/antibrane dynamics and KKLT stability,” [arXiv:1509.05710 \[hep-th\]](#).
- [182] J. P. Conlon, F. Quevedo, and K. Suruliz, “Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,” *JHEP* **08** (2005) 007, [arXiv:hep-th/0505076 \[hep-th\]](#).
- [183] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister, and S. P. Trivedi, “Towards inflation in string theory,” *JCAP* **0310** (2003) 013, [arXiv:hep-th/0308055 \[hep-th\]](#).
- [184] R. Blumenhagen, D. Herschmann, and E. Plauschinn, “The Challenge of Realizing F-term Axion Monodromy Inflation in String Theory,” *JHEP* **01** (2015) 007, [arXiv:1409.7075 \[hep-th\]](#).
- [185] S. Hosono, A. Klemm, and S. Theisen, “Lectures on mirror symmetry,” [arXiv:hep-th/9403096 \[hep-th\]](#). [Lect. Notes Phys.436,235(1994)].

BIBLIOGRAPHY

- [186] X. Dong, B. Horn, E. Silverstein, and A. Westphal, “Simple exercises to flatten your potential,” *Phys.Rev.* **D84** (2011) 026011, [arXiv:1011.4521 \[hep-th\]](#).